The Distribution of Prime Numbers on the Square Root Spiral

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contribution from " The Number Spiral " by

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Abstract :

Prime Numbers accumulate on defined spiral graphs, which run through the Square Root Spiral. These spiral graphs can be assigned to different spiral-systems, in which all spiral-graphs have the same direction of rotation and the same "second difference" between the numbers, which lie on these spiral-graphs. A mathematical analysis shows, that these spiral-graphs are caused exclusively by quadratic polynomials. For example the well known Euler Polynomial x^2+x+41 appears on the Square Root Spiral in the form of three spiral-graphs, which are defined by three different quadratic polynomials.

All natural numbers, divisible by a certain prime factor, also lie on defined spiral graphs on the Square Root Spiral (or "Spiral of Theodorus", or "Wurzelspirale"). And the Square Numbers 4, 9, 16, 25, 36 … even form a highly three-symmetrical system of three spiral graphs, which divides the square root spiral into three equal areas. Fibonacci number sequences also play a part in the structure of the Square Root Spiral. To learn more about these amazing facts, see my detailed introduction to the Square Root Spiral :

→ " **The ordered distribution of natural numbers on the Square Root Spiral** "

With the help of the "Number-Spiral" , described by Mr. Robert Sachs, a comparison can be drawn between the Square Root Spiral and the Ulam Spiral.

With the kind permission of Mr Robert Sachs, I show some sections of his webside : www.numberspiral.com in this study. These sections contain interesting diagrams, which are related to my analysis results, especially in regards to the distribution of prime numbers.

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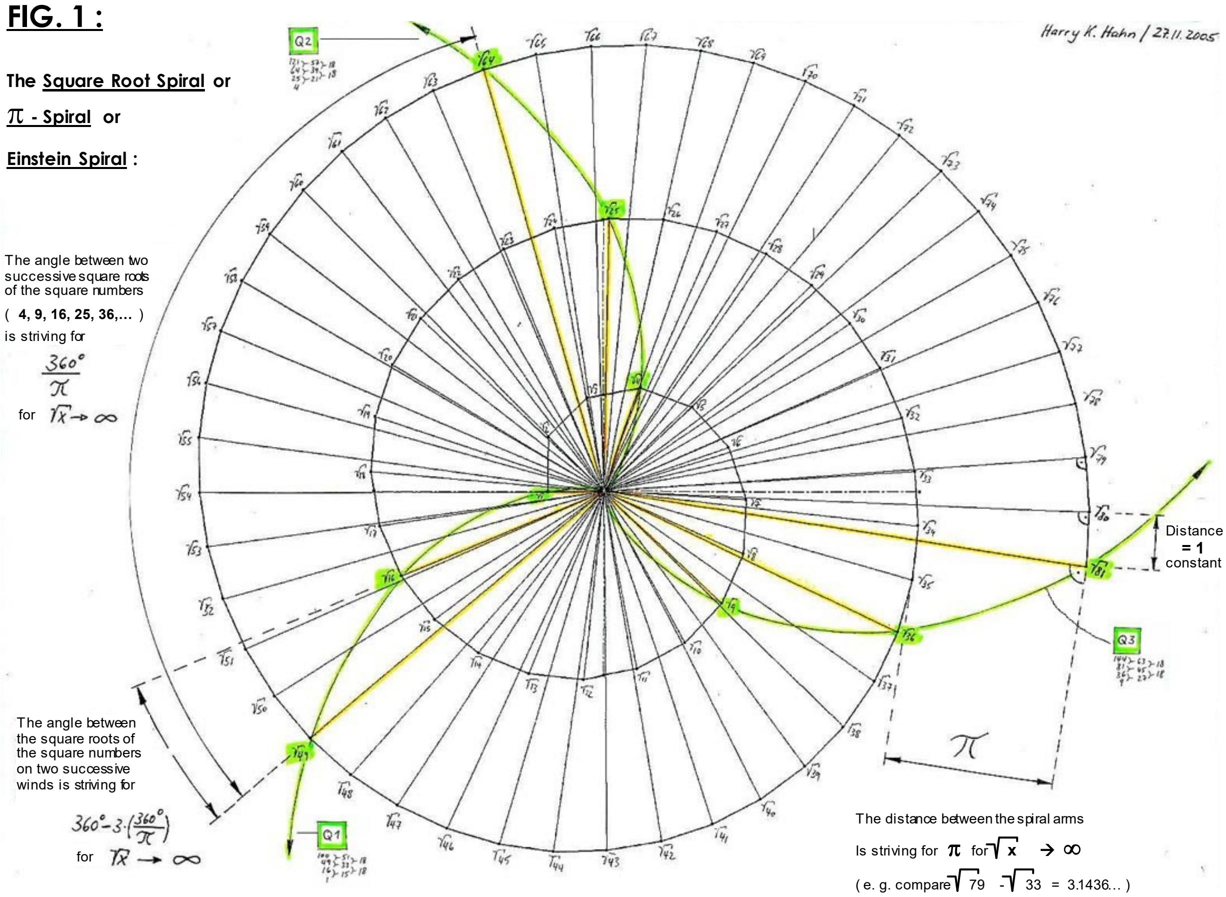
1 Introduction to the Square Root Spiral :

The Square Root Spiral (or “Spiral of Theodorus” or “Einstein Spiral”) is a very interesting geometrical structure in which the square roots of all natural numbers have a clear defined orientation to each other. This enables the attentive viewer to find many spatial interdependencies between natural numbers, by applying simple graphical analysis techniques. Therefore the Square Root Spiral should be an important research object for professionals, who work in the field of number theory !

Here a first impressive image of the Square Root Spiral :

FIG. 1 :



The most amazing property of the square root spiral is surely the fact, that the distance between two successive winds of the Square Root Spiral quickly strives for the well known geometrical constant **π** !!

Mathematical proof that this statement is correct is shown in **Chapter 1 “ The correlation to π “** in the mathematical section of my detailed introduction to the Square Root Spiral (→ previous study !) :

→ Title : “ The ordered distribution of the natural numbers on the Square Root Spiral ” → see ArXiv-achive

Another striking property of the Square Root Spiral is the fact, that the square roots of all square numbers (4, 9, 16, 25, 36…) lie on three highly symmetrical spiral graphs which divide the square root spiral into three equal areas. (→ see FIG.1 : graphs **Q1, Q2** and **Q3** drawn in green). For these three graphs the following rules apply :

1.) The angle between successive Square Numbers (on the “Einstein-Spiral”) is striving for $360°/\pi$ for sqrt(X) → ∞

2.) The angle between the Square Numbers on two successive winds of the “Einstein-Spiral” is striving for $360° - 3x(360°/\pi)$ for sqrt(X) → ∞

Proof that these propositions are correct, shows **Chapter 2 “ The Spiral Arms”** in the mathematical section of the above mentioned introduction study to the Square-Root Spiral.

The Square Root Spiral develops from a right angled base triangle (**P1**) with the two legs (cathets) having the length 1, and with the long side (hypotenuse) having a length which is equal to the square root of 2.

→ **see FIG. 2 and 3**

The square root spiral is formed by further adding right angled triangles to the base triangle **P1** (see FIG 3) In this process the longer legs of the next triangles always attach to the hypotenuses of the previous triangles. And the longer leg of the next triangle always has the same length as the hypotenuse of the previous triangle, and the shorter leg always has the length 1.
In this way a spiral structure is developing in which the spiral is created by the shorter legs of the triangles which have the constant length of 1 and where the lengths of the radial rays (or spokes) coming from the centre of this spiral are the square roots of the natural numbers (sqrt 2 , sqrt 3, sqrt 4, sqrt 5 ….).

→ **see FIG. 3**

The special property of this infinite chain of triangles is the fact that all triangles are also linked through the Pythagorean Theorem of the right angled triangle. This means that there is also a logical relationship between the imaginary square areas which can be linked up with the cathets and hypotenuses of this infinite chain of triangles (→ all square areas are multiples of the base area 1 , and these square areas represent the natural numbers N = 1, 2, 3, 4,…..) → **see FIG. 2 and 3**. This is an important property of the Square Root Spiral, which might turn out one day to be a "golden key" to number theory !

FIG. 2 :

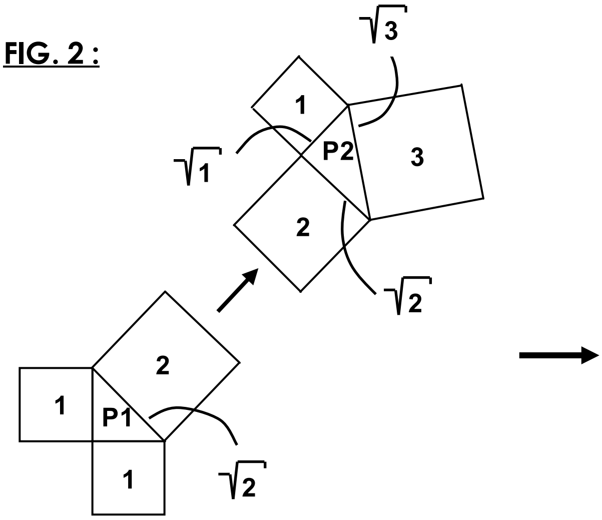
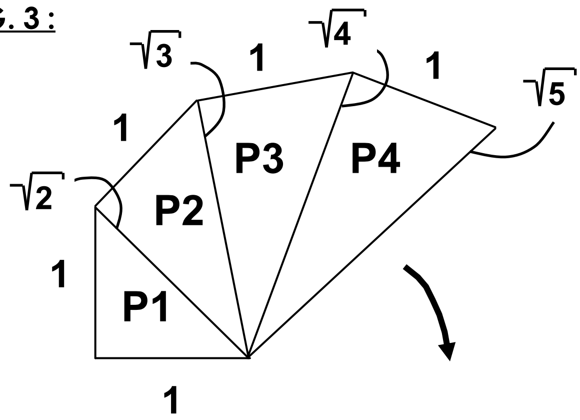


FIG. 3 :



By the way, the first two triangles **P1** and **P2** , which essentially define the structure of the complete Square Root Spiral ad infinitum, are also responsible for the definition of the cube structure. → **see FIG. 4**
Here the triangle **P1** defines the geometry of the area diagonal of the cube, whereas triangle **P2** defines the geometry of the space diagonal of the cube.

→ A cube with the edge length of **1** can be considered as base unit of space itself.

FIG. 4 :

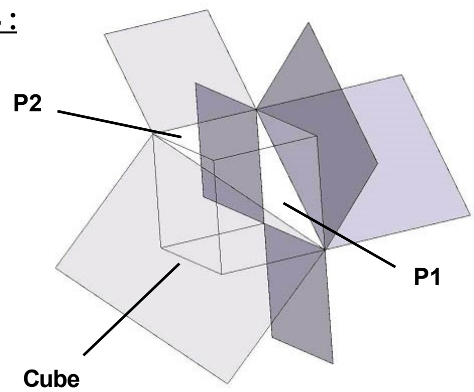
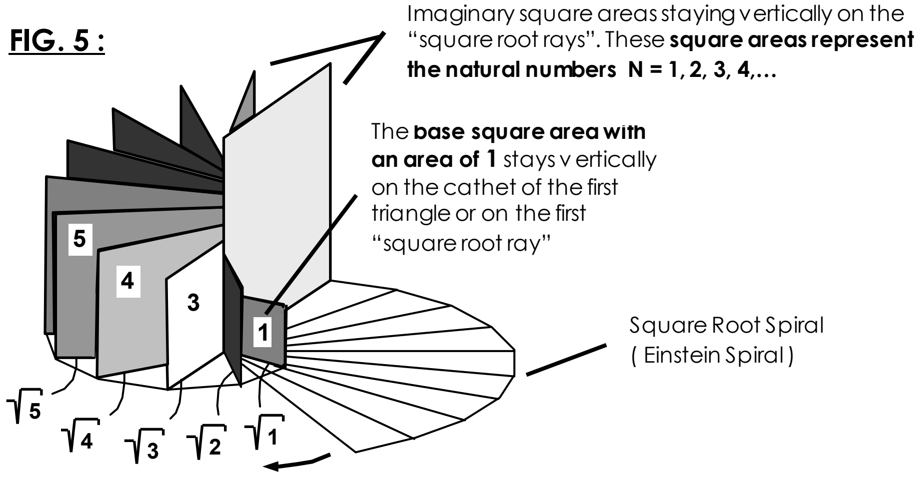


FIG. 1 shows the further development of the Square Root Spiral (or Einstein-Spiral) if one rectangular triangle after the other is added to the growing chain of triangles as described in FIG. 3.

For my further analysis I have created a square root spiral consisting of nearly 300 precise constructed triangles. For this I used the CAD Software SolidWorks. The length of the hypotenuses of these triangles which represent the square roots from the natural numbers 1 to nearly 300, has an accuracy of 8 places after the decimal point. Therefore, the precision of the square root spiral used for the further analysis can be considered to be very high.

The lengths of the radial rays (or spokes) coming from the centre of the square root spiral represent the square roots of the natural numbers (n = { 1, 2, 3, 4,...}) in reference to the length 1 of the cathets of the base triangle P1 (see FIG. 3). And the natural numbers themselves are imaginable by the areas of "imaginary squares", which stay vertically on these "square root rays". → **see FIG. 5** (compare with FIG.2)

FIG. 5 :

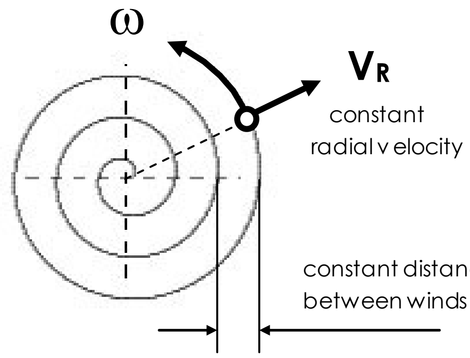


→ The "square root rays" of the Einstein Spiral can simply be seen as a projection of these spatial arranged "imaginary square areas", shown in FIG. 5, onto a 2-dimensional plane.

2 Mathematical description of the Square Root Spiral

Comparing the Square Root Spiral with different types of spirals (e.g. logarithmic-, hyperbolic-, parabolic- and Archimedes- Spirals), then the Square Root Spiral obviously seems to belong to the Archimedes Spirals.

An Archimedes Spiral is the curve (or graph) of a point which moves with a constant angle velocity around the centre of the coordinate system and at the same time with a constant radial velocity away from the centre. Or in other words, the radius of this spiral grows proportional to its rotary angle.



Archimedes Spiral

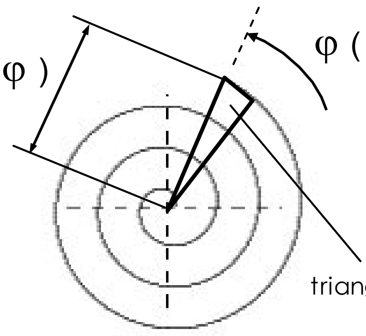
In polar coordinate style the definition of an Archimedes Spiral reads as follows :

$$r(\varphi) = a\varphi \qquad \text{with} \qquad a = \text{const.} = \frac{V_R}{\omega} > 0$$

for $r \rightarrow \infty$ the Square Root Spiral is an Archimedes Spiral with the following definition :

$$r(\varphi) = a\varphi + b$$

with a = const. and b = const.



The values of the parameters a and b are

$$a = \frac{1}{2} \quad \text{and} \quad b = -\frac{c_2}{2} \quad ; \quad \text{with} \quad c_2 = \text{Square Root Spiral Constant}$$

c_2 = - 2.157782996659….

Hence the following formula applies for the Square Root Spiral :

$$r(\varphi) = \frac{1}{2}\varphi + 1.078891498\ldots.. \qquad \text{for } r \rightarrow \infty$$

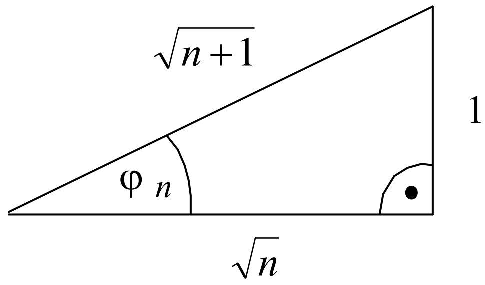
for $r \rightarrow \infty$ therefore the growth of the radius of the Square Root Spiral after a full rotation is striving for π (corresponding to the angle of a full rotation which is **2π**)

Note : The mathematical definitions shown on this page and on the following page can also be found either in the mathematical section of my introduction study to the Square Root Spiral
→ " The ordered distribution of natural numbers on the Square Root Spiral "
or in other studies referring to the Square Root Spiral.
→ e.g. a mathematical analysis of the Square Root Spiral is available on the following website : → **http://kociemba.org/themen/spirale/spirale.htm**

Further dependencies in the Square Root Spiral :

If φ_n is the angle of the ***n***th spiral segment (or triangle) of the Square Root Spiral, then

$\tan(\varphi_n) = \frac{1}{\sqrt{n}}$; (ratio $\frac{\text{counter cathet}}{\text{cathet}}$)



If the ***n***th triangle is added to the Square Root Spiral the growth of the angle is

$$\varphi_n = \arctan\left(\frac{1}{\sqrt{n}}\right)$$; <u>Note</u> : angle in radian

The total angle $\varphi(k)$ of a number of **k** triangles is

$$\varphi(k) = \sum_{n=1}^{k} \varphi_n$$ or described by an integral $$\int_0^k \arctan\left(\frac{1}{\sqrt{n}}\right) dn + c_1(k)$$

$$\Rightarrow \quad \varphi(k) = 2\sqrt{k} + c_2(k) \quad \text{with} \quad \lim_{k\to\infty} c_2(k) = \text{const.} = -2.157782996659.....$$

c_2 = Square Root Spiral Constant

The growth of the radius of the Square Root Spiral at a certain triangle n is

$$\Delta r = \sqrt{n+1} - \sqrt{n}$$

The radius r of the Square Root Spiral (i.e. the big cathet of triangle **k**) is

$r(k) = \sqrt{k}$ and by converting the above shown equation for $\varphi(k)$ it applies that

$$r(k(\varphi)) = r(\varphi) = \sqrt{\frac{1}{4}(\varphi - c_2(\varphi))^2} = \frac{1}{2}\varphi - \frac{c_2}{2}$$

For large n it also applies that φ_n is approximately $\frac{1}{\sqrt{n}}$ and Δr has pretty well half of this value, that is $\frac{1}{2\sqrt{n}}$, what can be proven with the help of a Taylor Sequence.

3 The distribution of Prime Numbers on defined spiral-graphs :

In a similar way as the Square Numbers shown in **FIG.1** , Prime Numbers also accumulate on certain spiral-graphs, which run through the Square Root Spiral (Einstein Spiral).

After marking all Prime Numbers with yellow color on the Square Root Spiral , it is easy to see, that the Prime Numbers accumulate on certain spiral-shaped graphs. I want to call these spiral-graphs with a high share in prime numbers "Prime Number Spiral-Graphs".

To identify these "Prime Number Spiral-Graphs", I marked the most conspicuous spiral graphs with a high share of Prime Numbers, then I tried to make sense out of their arrangement on the Square Root Spiral .

→ **see FIG. 6-A** to **6-B** on the following pages.

A closer look to these " Prime Number Spiral-Graphs" revealed the following properties :

- It is obvious that certain Prime Number Spiral-Graphs are related to each other and that they belong to the same "spiral-graph-system".
- By calculating the differences (first difference) of the consecutive numbers lying on one of the found "Prime Number Spiral-Graphs", and by further calculating the differences of these differences (second difference) we always obtain one of the following three numbers :

  18, 20 or **22** → I called these numbers the " **2. Differential** " of the spiral graphs.

  → see for example the difference calculation in FIG. 6-A for the exemplary spiralarm **A3** : (see PNS-P18-A)

  The calculation of the differences between the numbers 11, 41, 89, 155, 239,…. , which lie on the spiralarm A3 results in the following numbers : 30, 48, 66, 84,….. And the calculation of the differences between these numbers results in the constant value **18**.
  And this number represents the " **2. Differential** " of this spiralarm **A3**.

It is notable that the 2. Differential of the Prime Number Spiral-Graphs is always an even number .
And it seems that the 2. Differential only takes on one of these three values : **18, 20** or **22**

That is the reason why I used these three different possible values of the **2. Differential** as distinguishing property for the graphical representation of the Prime Number Spiral-Graphs shown in FIG. 6-A to 6-C.

→ Therefore in **FIG. 6-A to 6-C** the following assignment applies :

FIG. 6-A : shows only Prime Number Spiral-Graphs which have a **2. Differential** of **18**

FIG. 6-B : shows only Prime Number Spiral-Graphs which have a **2. Differential** of **20**

FIG. 6-C : shows only Prime Number Spiral-Graphs which have a **2. Differential** of **22**

3.1 The found Spiral-Graphs can be assigned to different Spiral-Graph-Systems

In my attempt to make sense out of the distribution of the Prime-Numbers on the Square Root Spiral, I first tried to establish order under the found Prime Number Spiral Graphs.

By doing this, I realized that the Prime Number Spiral Graphs are arranged in different " systems" .

As best example I want to refer to **FIG. 6-A → see following pages !**

→ **FIG. 6-A** : The **3** Prime Number Spiral Systems shown in this diagram all have a **2. Differential** of **18** !!

→ see difference calculation for the three exemplary spiralarms **A3, B5** and **C12**

The diagram shows how the Prime Numbers are clearly distributed on **3** defined spiral graph systems, which are arranged in a highly symmetrical manner (in an angle of around 120° to each other) around the centre of the Square Root Spiral.

On the shown **3 Prime Number-**(**Spiral**)**-Systems** (**PNS**) : **P18-A, P18-C** and **P18-C** , the Prime Numbers are located on pairs of spiral arms, which are separated by three numbers in between. And two spiral arms of one such pair of spiral arms, are separated by one number in between.

All spiral-graphs of the shown **3** Prime Number-(Spiral)-Systems (**PNS**) have a **positive rotation direction** (**P**) and, as already mentioned before, the **2. Differential** of all spiral-graphs has the constant value of **18**. That's why the first part of the naming of the **3** Prime Number-Spiral-Systems (**PNS**) is **P18**.
The **3** spiral-graph systems **A** (drawn in orange), **B** (drawn in pink) and **C** (drawn in blue) have further spiralarms. But for clearness there are only around 10 spiralarms drawn per system.

One important property of all Prime Number spiral-graphs (shown in FIG 6-A) is the obvious missing of numbers which are divisible by **2** or **3** in these graphs ! That means, that the smallest possible prime factor of the " Non-Prime Numbers" , which lie on these spiralarms, is **5**.

→ **FIG. 6-B** , on the following pages, shows a diagram with another set of **12** Prime Number Spiral Systems In this diagram all Spiral-Graphs have a **2. Differential** of **20** !!

→ see difference calculation for the four exemplary spiralarms **D8, F2, G5** and **I5**

On the shown **12 Prime Number Spiral Systems** (**PNS**) : **N20-D** to **N20-I** , and **P20-D** to **P20-I** , the Prime Numbers are again located on pairs of spiral arms, which are separated by three number in between. And two spiral arms of one such pair of spiral arms, are again separated by one number in between.

6 of the shown Prime-Number-Spiral-Systems (**PNS**) have a **positive rotation direction** (**P**) and the other **6** Prime-Number-Spiral-Systems (**PNS**) have a **negative rotation direction** (**N**). The spiral-graph systems of these two groups are arranged in a symmetrical manner around the centre of the Square Root Spiral, in an angle of approx. 60° to each other. And two systems at a time are approx. point-symmetrical to each other (in reference to the centre of the Square Root Spiral). For example the two systems **N20-I** & **N20-F**

For clearness only **4** Prime-Number-Spiral-Systems (**PNS**) with a **negative rotation direction** (**N**) are drawn in color !! These are the **4** systems : **N20-D** (drawn in orange) , **N20-F** (drawn in red) , **N20-G** (drawn in blue), **N20-I** (drawn in pink). There are only around 10 spiralarms drawn ofr each of these 4 spiral systems.

Note, that the **other 8** Prime Number Spiral Systems are all drawn in light grey color, and that there are only 2 to 6 spiralarms drawn of each of these systems, for clearness ! Please also note, that only the Spiral-Graphs with a **negative rotation direction** (**N**) are named in FIG 6-B. The naming of the Spiral-Graphs with a **positive rotation direction** (**P**) P-20… would in principle just mirror the naming of the "N20…-spiralarms". → For example the spiralarm with the " P20-G" -mark attached to it (= name of this system), would be named G1 and the next spiralarm on the left G2 and so on.

→ **FIG. 6-C**, shows the third group of Spiral-Systems , which contains altogether **11** Prime Number Spiral Systems , which all have a **2. Differential** of **22** !!

→ see difference calculation for the 4 exemplary spiralarms **J10, L6, N11** and **Q7**

On the shown **11 Prime Number Spiral Systems** (**PNS**) : **N22-J** to **N22-T** , the Prime Numbers are again located on pairs of spiralarms, which are separated by three numbers in between. And two spiralarms of one such pair of spiralarms, are again separated by one number in between.

All **11** shown Prime Number Spiral Systems (**PNS**) have a **negative rotation direction** (**N**). And it seems that the spiral graph systems are arranged in a symmetrical manner around the centre of the Square Root Spiral (in an angle of ~ 360°/11 to each other, in reference to the centre of the Square Root Spiral).

For clearness only **4** Prime Number Spiral Systems (**PNS**) are drawn in color !! These are the following **4** systems : **N22-J** (drawn in orange) , **N22-L** (drawn in pink), **N22-N** (drawn in blue) , **N20-Q** (drawn in red). Note, that there are only around 10 spiralarms drawn for each of these four spiral-systems.

Note, that the **other 7** Prime Number Spiral Systems are all drawn in light grey color, and that there are only around 4 to 6 spiralarms drawn of each of these systems !
For clearness, the spiralarms of these 7 systems are not named on FIG. 6-C. But the naming of these spiralarms would start on the spiralarm which has the naming of the system attached to it. For example the spiralarm with the " N22-M" -mark attached to it would be named M1, and the next spiralarm below would be named M2 and so on.

FIG. 6-A :

Prime Number Spiral-Graphs
with " 2. Differential " = 18

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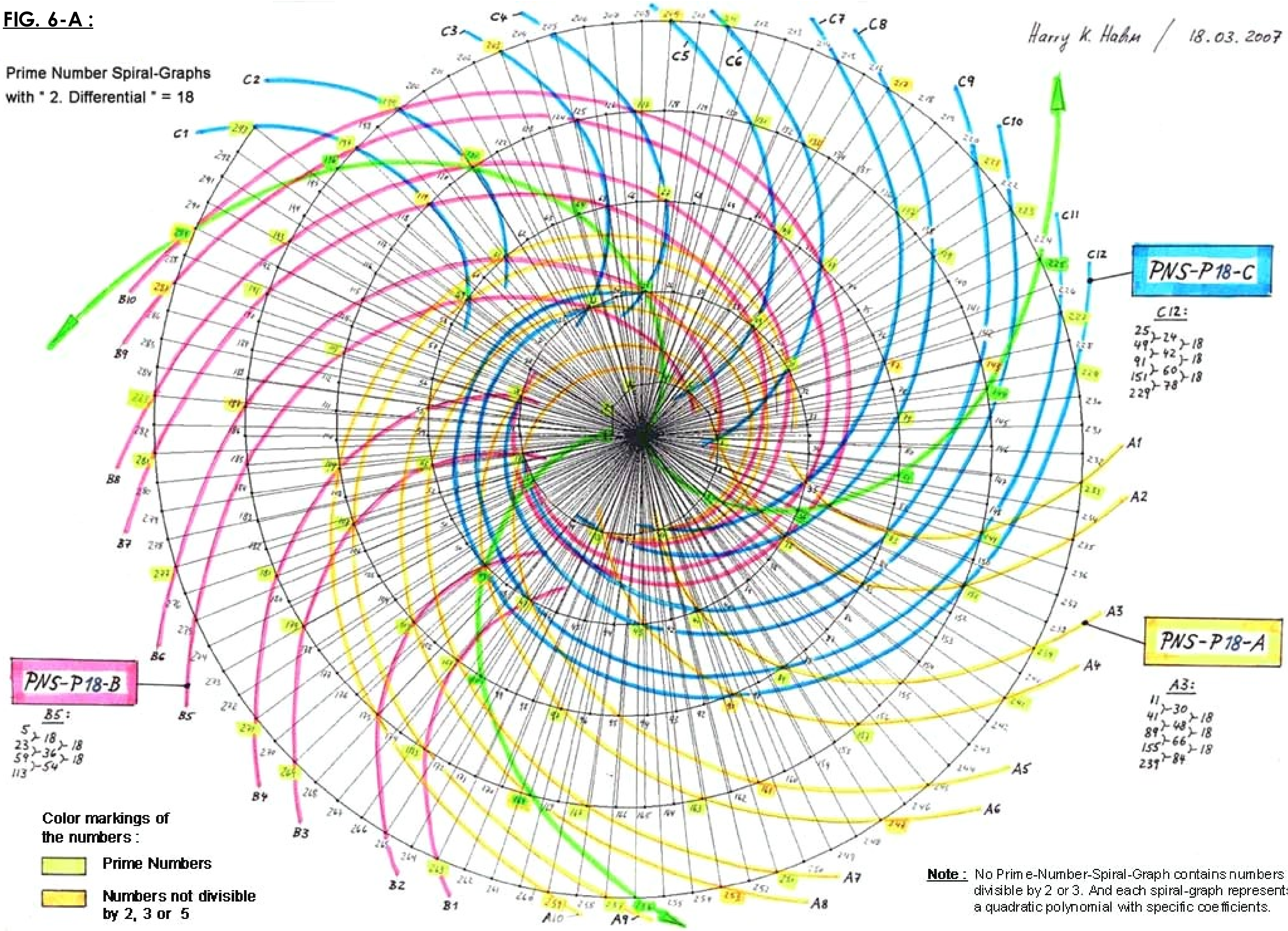
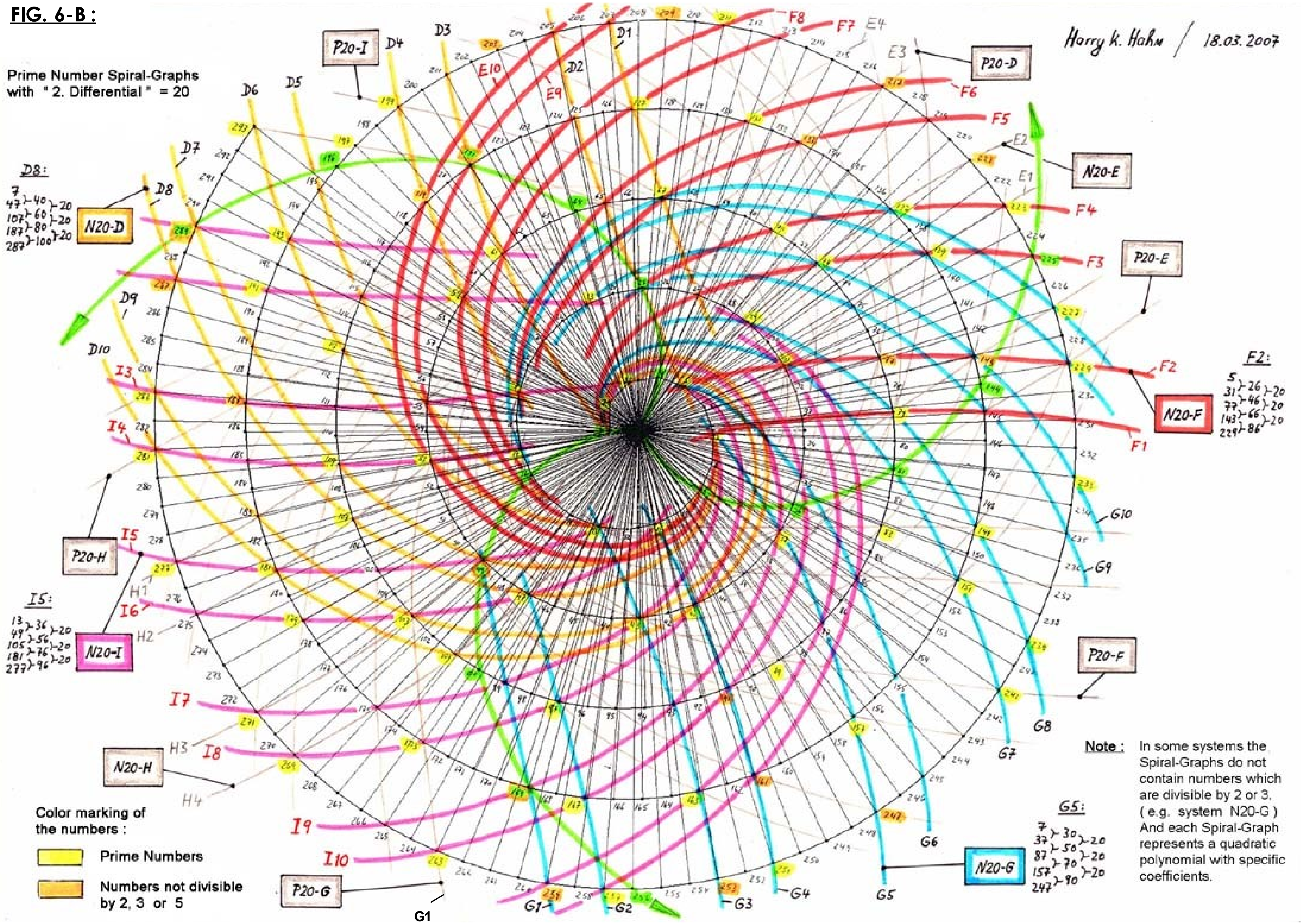


Color markings of the numbers :

- Prime Numbers
- Numbers not divisible by 2, 3 or 5

Note : No Prime-Number-Spiral-Graph contains numbers divisible by 2 or 3. And each spiral-graph represents a quadratic polynomial with specific coefficients.

FIG. 6-B :



Prime Number Spiral-Graphs
with " 2. Differential " = 20
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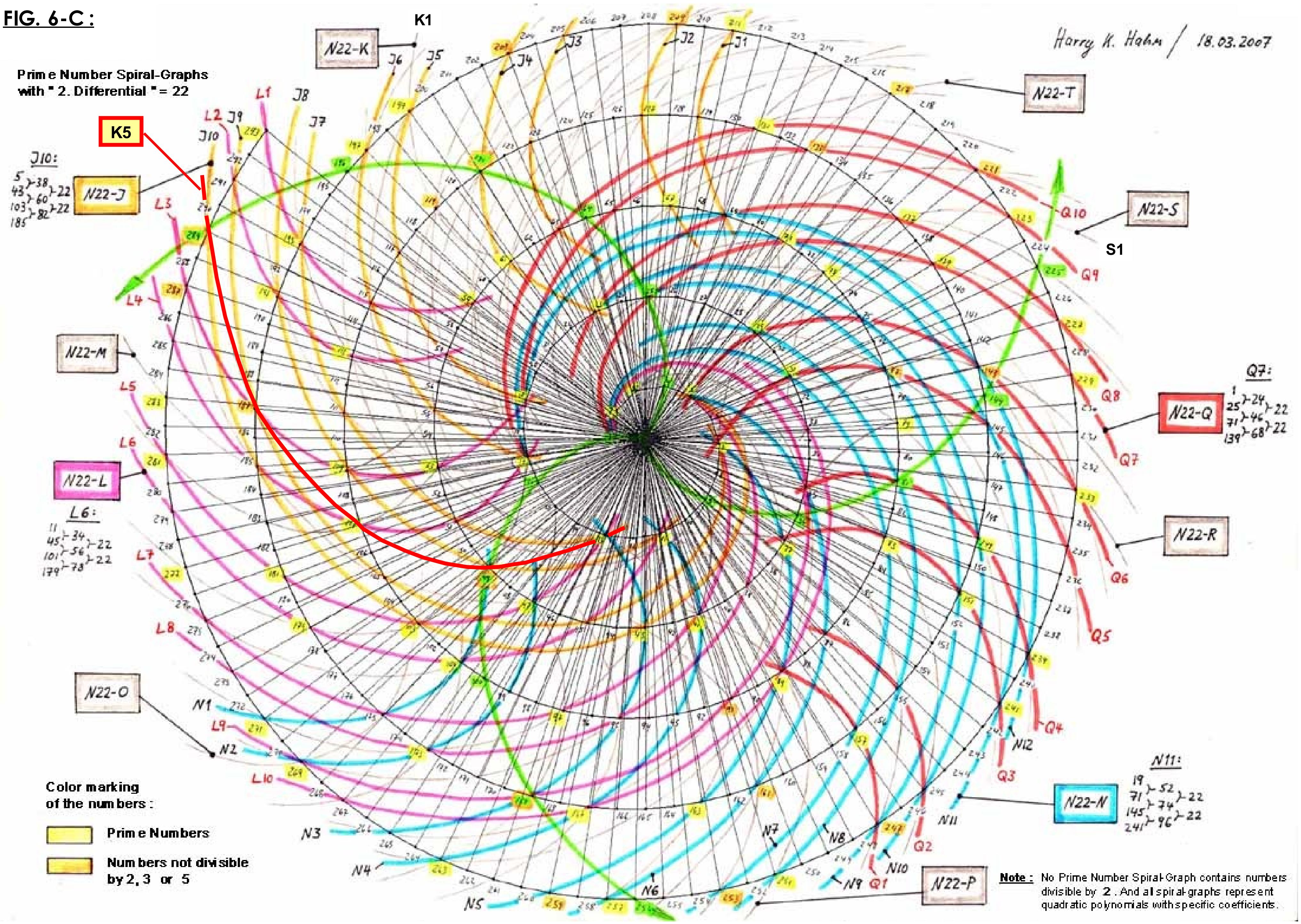
P20-I
P20-D
N20-E
P20-E
N20-F
P20-F
N20-G
P20-G
N20-H
P20-H
N20-I
N20-D
D8:
7
47 } 40 } 20
107 } 60 } 20
187 } 80 } 20
287 } 100 } 20
F2:
5
31 } 26 } 20
77 } 46 } 20
143 } 66 } 20
229 } 86
I5:
13 } 36 } 20
49 } 56 } 20
105 } 76 } 20
181 } 96 } 20
277
G5:
7 } 30 } 20
37 } 50 } 20
87 } 70 } 20
157 } 90
247
Color marking of the numbers :
Prime Numbers
Numbers not divisible by 2, 3 or 5
Note : In some systems the Spiral-Graphs do not contain numbers which are divisible by 2 or 3. (e.g. system N20-G) And each Spiral-Graph represents a quadratic polynomial with specific coefficients.
G1

FIG. 6-C :


Prime Number Spiral-Graphs
with " 2. Differential " = 22
Harry K. Hahn / 18.03.2007
K1
K5
S1
N22-K
N22-T
N22-J
N22-S
N22-M
N22-Q
N22-L
N22-R
N22-O
N22-N
N22-P
J10:
L6:
Q7:
N11:
Color marking
of the numbers :
Prime Numbers
Numbers not divisible
by 2, 3 or 5
Note : No Prime Number Spiral-Graph contains numbers divisible by 2 . And al spiral-graphs represent quadratic polynomials with specific coefficients.

4 Number Sequences derived from the Spiral-Graphs shown in FIG. 6-A to 6-C

In the following section I want to show an analysis of the Number Sequences which belong to the " Prime Number Spiral Graphs" shown in FIG 6-A to 6-C.

In the Tables **6-A1** to **6-C1** which refer to the diagrams 6-A to 6-C, I have set-up three columns for each analysed "Prime Number Spiral Graph".
(→ Tables **6-B1** and **6-C1** can be found **in the Appendix** !)

The three colums in Table **6-A1**, which refer to the exemplary Spiral-Graph **A3** (→ see **FIG 6-A**) are named A3, A3' and A3''.

In the left column A3 , I listed all " known" numbers which lie on this spiral-graph (see FIG 6-A). These are the numbers 11, 41, 89, 155 and 239.

For each analysed Spiral-Graph, there are 4 to 6 numbes available, which can be extracted from the graphs drawn in FIG 6-A to FIG 6-C.

With the help of at least three numbers of each Prime Number Spiral Graph the "first differences" and the "second differences" (2. Differential) of these numbers can be calculated, which then can be used for the further development of the number sequence belonging to the analysed spiral-graph.

For the exemplary spiral-graph A3, the "first differences" between the known five numbers 11, 41, 89, 155, 239 are 30, 48, 66, 84.
And the "second differences" (2. Differential) for this number sequence result in the constant value 18.

Now the "first differences" in column A3' can be extended, with the help of the calculated 2. Differential = A3'' = 18. And with the help of the extended sequence A3' the number sequence of the Prime Number Spiral Graph A3 can be further extended too.

In this way, I have extended the number-sequences derived from the spiral-graphs shown in FIG 6-A to FIG 6-C up to numbers > 100 000.

4.1 Quadratic Polynomials are the foundation of the Spiral-Graphs :

The calculated " first differences" and " second differences" can also be used to determine the quadratic polynomials which define these Prime Number Spiral Graphs. An explanation of the calculation procedure (→ Newton Interpolation Polynomial), to determine the quadratic polynomials of the Prime Number Spiral Graphs, is available in my introduction to the Square Root Spiral :

- → **"The ordered distribution of the natural numbers on the Square Root Spiral"**
- → This study can be found under my author-name in the arXiv - archive

Please also have a read through this study ! There is a mathematical section included in this study, which describes the shown Spiral-Graphs from the mathematical point of view.

The calculated **quadratic polynomials**, which define the Prime Number Spiral Graphs, and the number sequences belonging to it, can be found in the **Tables 6-A2** to **6-C2** → **see next pages (Table 6-A2)**
and Appendix (Table 6-B2 and Table 6-C2) !

→ Referring to the general quadratic polynomial : $f(x) = \mathbf{ax^2 + bx + c}$

the following rules apply for the quadratic polynomials, belonging to the analysed spiral-graphs (→ shown in Tables 6-A2 to 6-C2) :

Rules for coefficients **a, b** and **c** :
(or meaning of coefficients)

- **a** → equivalent to the **2.Differential** of the Spiral-Graph divided by **2**
- **b** → this coefficient (or sequence of coefficients), refers to the system the Spiral-Graph is belonging to.
- **c** → consecutive parallel distance between the Spiral-Graphs , belonging to the same system.

The next step in my analysis of the number sequences shown in the tables 6-A1 to 6-C1, was the marking of the Prime Numbers in the first 25 numbers of each sequence. I used the color yellow to mark the Prime Numbers. I also marked the numbers which are not divisible by 2, 3 or 5 in the first 25 numbers of these sequences with red color.

Additional to the described three columns per number-sequence , I added another column to each analysed sequence (→ see Tables 6-A1 to 6-C1).
This additional column has the naming " SD " , which means " sum of the digits" and it shows the sums of the digits of the first 25 numbers of each number sequence.
For example the " sums of the digits" of the first 3 numbers of the A3 - number sequence are 2 (= 1 + 1 → 11) ; 5 (= 4 + 1 → 41) ; 17 (= 8 + 9 → 89) etc.

By looking over tables 6-A1 to 6-C1 , it is easy to see, that there are differences in the distribution of the color marked cells, and that in many of the shown number sequences, the distribution of the color-marked cells has a clear periodic character !

Now we want to start with a more detailed analysis of the number sequences shown in the Tables 6-A1, 6-B1 and 6-C1 :

4.2 Differences in the Number-Sequences shown in Table 6-A1 to 6-C1

→ **Here is a list of some differences in these number-sequences, which are easy to notice :**

- Approximately **30 %** of all analysed number sequences only contain numbers which are marked in yellow and red. This means they do not contain numbers which are divisible by 2, 3 or 5.
  Therefore all non-prime numbers (marked in red) in these number sequences consist of prime factors ≥ 7
- It is easy to see, that the white cells and the colored cells (yellow or red) periodically alternate in various ways, in the most number-sequences.
- A closer look shows, that there are sequences, where numbers divisible by 3 or 5 occur in a periodic manner.
  For example in the number-sequences A3, A10 and B4 (in Table 6-A1) a number divisible by 5 periodically alternates with 4 numbers which are not divisible by 2, 3 or 5. Or in the number-sequences K2, L2 and M2 (see Table 6-C1 in the Appendix) a number divisible by 3 periodically alternates with two numbers not divisible by 2, 3, or 5 .
- All Spiral-Graphs of the spiral-graph systems N20-D, N20-G, P20-D and P20-G (→ see FIG 6-B) only contain numbers with the same ending. For example the Spiral-Graph **D8** in FIG 6-B only contains numbers with the ending **7**.
- The number sequences of all other spiral-graphs in FIG 6-A to FIG 6-C contain numbers, whose endings always follow a sequence of exactly 5 numbers, which is periodically recurring ad infinitum.

  For example Spiral-Graph **Q3** (→ see FIG 6-C and Table 6-C1) only contains numbers with the ending 1, 3 or 7. And these number-endings always occur in the following sequence : **…3, 1, 1, 3, 7,…..**, which is recurring in a periodic manner.

In principle these were the most noticeable differences of the number-sequences listed in Table 6-A1 to 6-C1.

4.3 Analysis of the number-endings :

A closer look shows, that the number-endings, which occur in the number-sequences depend on the "2.Differential" of the number-sequence (or spiral-graph). This connection is shown in **Table 3** (→ see next pages).

In this table the prime number sequences (shown in table 6-A1 to 6-C1) are ordered according to the number-endings, which occur in these sequences.

It is easy noticeable that in number-sequences, which have the 2. Differential 18 or 22 (see first column of table), the number-endings are defined by the same groups of 3 numbers.

For example the number-endings 1, 5, 7 or 1, 3, 7 occur in number-sequences which have either the 2. Differential 18 or 22.

The only differences are the "Number-Endings-Sequences" in which these groups of three numbers occur !

For example in number-sequences which have the 2. Differential 18 , the number-endings 1, 5, 7 or 1, 3, 7 occur as "Number-Endings-Sequences" …7, 7, 5, 1, 5,… and …3, 3, 1, 7, 1,…., whereas the same number-endings occur as quite different number-endings-sequences …7, 5, 5, 7, 1,… and …3, 1, 1, 3, 7,….. in the number-sequences with the 2. Differential 22.

In number-sequences which have the 2. Differential 20 all numbers either have the same ending, which is one of the odd numbers 1, 3, 5, 7 or 9 , or the numbers have as endings the mentioned 5 odd numbers all in the same sequence. In number-sequences, where all 5 odd numbers occur, one of the following two number-endings-sequences appears : …1, 5, 9, 3, 7,…. or …7, 3, 9, 5, 1,…. → Noticeable is the **opposite** direction of these two sequences !

The most remarkable property of all number-endings-sequences listed in Table 3, is certainly the fact, that all number-endings-sequences cover exactly **5** consecutive numbers !

This indicates, that there is a kind of "higher basic-oscillation" acting in the Square Root Spiral, which seems to cover exactly 5 windings of the Square Root Spiral per oscillation, and which interacts with all Prime Number Spiral Graphs shown in FIG 6-A to 6-C ! This amazing fact is worth an own analysis !!

From the mathematical point of view the following explanation can be given for the periodic occurrence of the number-endings described in Table 3 :

"…… Because of the recursive representation

$$p(t+1) = p(t) + a(2t+1) + b = f(p(t), t)$$

the sequence (***p(t)*** mod *k*) recurs for each natural *k*, since only a maximum of k^2 pairs (***p(t), t***) are available. This explains on the one hand the recurrence of the last sequence of figures (*k* = 10) and on the other hand the regular occurrence of certain *k* factors (as shown e.g. in Table 4, 5-A & 5-B).
This applies to all *k*, not only to the prime numbers.
The sequence of numbers not divisible by 2, 3 or 5 is recurringly modulo 30 and hence a repetend of the corresponding figures also exists in the series (number-sequences) shown in Table 6-A1to 6-C1.
The length of this repetend can theoretically be $30^2 = 900$ ……"

This mathematical explanation for the periodic occurrence of the number-endings shown in Table 3 is from Mr. Kay Schoenberger, who also contributed the mathematical section of my introduction to the Square Root Spiral.

→ **"The ordered distribution of the natural numbers on the Square Root Spiral"**
(→ This study can be found under my author-name in the arXiv – data bank)

Please have a read through the mathematical section included in this study, which describes the shown spiral-graphs from the mathematical point of view.

Table 3 gives a general information about the periodic occurrence of numbers divisible by 3 or 5 in the number-sequences derived from the Prime Number Spiral Graphs → see last two columns on the righthand side of the table .

4.4 Analysis of the "sums of the digits" :

Another remarkable property of the number-sequences listed in Table 6-A1 to 6-C1 is the fact, that only defined "sums of the digits" occur in every number-sequence. In the following we want to carry out a simple analysis of the "sums of the digits" which occur in these number sequences.

As described before, I added a column with the naming " SD " (which means " sum of the digits") to each number-sequence listed in Table 6-A1 to 6-C1. And the numbers in this column represent the sums of the digits of the first 25 numbers of each number sequence. → For example the "sums of the digits" of the first three numbers of the **A3** – number-sequence in Table 6-A1 are **2** (= 1 + 1 → 11) ; **5** (= 4 + 1 → 41) ; **17** (= 8 + 9 → 89) etc.

By looking over the numbers listed in the columns "SD" it is easy to see, that there are only defined "SD" - numbers occurring in each number-sequence.

And by putting these sums of the digits **in order** (according to their value !) , a certain "sums of the digits – sequence" for every number-sequence in Table 6-A1 to 6-C1 can be found.

Table 2 on the next pages shows the results of this analysis !

In this table the number-sequences (shown in table 6-A1 to 6-C1) are ordered according to the "sums of the digits – sequences" , which occur in these sequences.

Here the following classification can be made :

Number-sequences with the 2. Differential **18** produce sums of the digits – sequences with either the distance of **3** or **9** between two consecutive numbers of the sums of the digits – sequence.

For example the number-sequences A2, A4, A6,…etc. produce the sums of the digits – sequence 4, 7, 10, 13, 16,… in which the distance between two consecutive numbers is **3**. Or the number-sequences B1, B7, B13,…, as another example, produce the sums of the digits – sequence 2, 11, 20, 29, 38,… in which the distance between two consecutive numbers is **9**.
Number-sequences with the 2. Differential 20 or 22 produce "sums of the digits" – sequences which show a kind of periodic behavior similar to the "number-endings-sequences" in Table 3.

Here the differences between **5** consecutive numbers of the sums of the digits – sequences form a periodic sequence of four numbers which recur ad infinitum.

For example the number-sequences N20-D1 and N20-F3 with the 2. Differential **20**, produce the following **ordered** sums of the digits – sequence : 1, 4, 7, 9, 10, 13, 16, 18, 19,… in which the periodic recurring differences **…..3, 3, 2, 1,….** occur.

And as another example, the number-sequences K3 and L1 with the 2. Differential **22**, produce the following **ordered** sums of the digits – sequence : 4, 5, 7, 10, 13, 14, 16, 19, 22,..… in which the periodic recurring differences **…..1, 2, 3, 3,….** occur.

Remarkable is here the fact, that the direction of the periodic recurring differences …..3, 3, 2, 1,…. and …..1, 2, 3, 3,…. in the sums of the digits – sequences, is exactly opposite in the number-sequences with the different "2. Differentials" 20 or 22.

But even more remarkable is the fact, that these mentioned "periodic-recurring-difference-sequences" only occur in the **ordered** sums of the digits – sequences shown in Table 2, but no periodic behavior at all, of the **unordered** " sums of the digits" can be noticed in the "SD" - colums in Table 6-A1 to 6-C1 !!

I haven't found an explanation for this strange characteristic yet !

Table 6-A1 : "Prime Number Sequences" derived from the graphs shown in the "Prime Number Spiral Systems" **P18-A ; P18-B** and **P18-C** → see **FIG. 6-A**

SD	A1	A1'	A1"	SD	A2	A2'	A2"	SD	A3	A3'	A3"	SD	A4	A4'	A4"
5	5			7	7			2	11			4	13		
8	35	30		10	37	30		5	41	30		7	43	30	
11	83	48	18	13	85	48	18	17	89	48	18	10	91	48	18
14	149	66	18	7	151	66	18	11	155	66	18	13	157	66	18
8	233	84	18	10	235	84	18	14	239	84	18	7	241	84	18
11	335	102	18	13	337	102	18	8	341	102	18	10	343	102	18
14	455	120	18	16	457	120	18	11	461	120	18	13	463	120	18
17	593	138	18	19	595	138	18	23	599	138	18	7	601	138	18
20	749	156	18	13	751	156	18	17	755	156	18	19	757	156	18
14	923	174	18	16	925	174	18	20	929	174	18	13	931	174	18
8	1115	192	18	10	1117	192	18	5	1121	192	18	7	1123	192	18
11	1325	210	18	13	1327	210	18	8	1331	210	18	10	1333	210	18
14	1553	228	18	16	1555	228	18	20	1559	228	18	13	1561	228	18
26	1799	246	18	10	1801	246	18	14	1805	246	18	16	1807	246	18
11	2063	264	18	13	2065	264	18	17	2069	264	18	10	2071	264	18
14	2345	282	18	16	2347	282	18	11	2351	282	18	13	2353	282	18
17	2645	300	18	19	2647	300	18	14	2651	300	18	16	2653	300	18
20	2963	318	18	22	2965	318	18	26	2969	318	18	19	2971	318	18
23	3299	336	18	7	3301	336	18	11	3305	336	18	13	3307	336	18
17	3653	354	18	19	3655	354	18	23	3659	354	18	16	3661	354	18
11	4025	372	18	13	4027	372	18	8	4031	372	18	10	4033	372	18
14	4415	390	18	16	4417	390	18	11	4421	390	18	13	4423	390	18
17	4823	408	18	19	4825	408	18	23	4829	408	18	16	4831	408	18
20	5249	426	18	13	5251	426	18	17	5255	426	18	19	5257	426	18
23	5693	444	18	25	5695	444	18	29	5699	444	18	13	5701	444	18
17	6155	462	18	19	6157	462	18	14	6161	462	18	16	6163	462	18
20	6635	480	18	22	6637	480	18	17	6641	480	18	19	6643	480	18

SD	A5	A5'	A5"	SD	A6	A6'	A6"	SD	A7	A7'	A7"	SD	A8	A8'	A8"
5	5			7	7			2	11			4	13		
8	17	12		10	19	12		5	23	12		7	25	12	
11	47	30	18	13	49	30	18	8	53	30	18	10	55	30	18
14	95	48	18	16	97	48	18	2	101	48	18	4	103	48	18
8	161	66	18	10	163	66	18	14	167	66	18	16	169	66	18
11	245	84	18	13	247	84	18	8	251	84	18	10	253	84	18
14	347	102	18	16	349	102	18	11	353	102	18	13	355	102	18
17	467	120	18	19	469	120	18	14	473	120	18	16	475	120	18
11	605	138	18	13	607	138	18	8	611	138	18	10	613	138	18
14	761	156	18	16	763	156	18	20	767	156	18	22	769	156	18
17	935	174	18	19	937	174	18	14	941	174	18	16	943	174	18
11	1127	192	18	13	1129	192	18	8	1133	192	18	10	1135	192	18
14	1337	210	18	16	1339	210	18	11	1343	210	18	13	1345	210	18
17	1565	228	18	19	1567	228	18	14	1571	228	18	16	1573	228	18
11	1811	246	18	13	1813	246	18	17	1817	246	18	19	1819	246	18
14	2075	264	18	16	2077	264	18	11	2081	264	18	13	2083	264	18
17	2357	282	18	19	2359	282	18	14	2363	282	18	16	2365	282	18
20	2657	300	18	22	2659	300	18	17	2663	300	18	19	2665	300	18
23	2975	318	18	25	2977	318	18	20	2981	318	18	22	2983	318	18
8	3311	336	18	10	3313	336	18	14	3317	336	18	16	3319	336	18
20	3665	354	18	22	3667	354	18	17	3671	354	18	19	3673	354	18
14	4037	372	18	16	4039	372	18	11	4043	372	18	13	4045	372	18
17	4427	390	18	19	4429	390	18	14	4433	390	18	16	4435	390	18
20	4835	408	18	22	4837	408	18	17	4841	408	18	19	4843	408	18
14	5261	426	18	16	5263	426	18	20	5267	426	18	22	5269	426	18
17	5705	444	18	19	5707	444	18	14	5711	444	18	16	5713	444	18
20	6167	462	18	22	6169	462	18	17	6173	462	18	19	6175	462	18

SD	A9	A9'	A9"	SD	A10	A10'	A10"	SD	A11	A11'	A11"	SD	A12	A12'	A12"
8	17			10	19			5	23			7	25		
11	29	12		4	31	12		8	35	12		10	37	12	
14	59	30	18	7	61	30	18	11	65	30	18	13	67	30	18
8	107	48	18	10	109	48	18	5	113	48	18	7	115	48	18
11	173	66	18	13	175	66	18	17	179	66	18	10	181	66	18
14	257	84	18	16	259	84	18	11	263	84	18	13	265	84	18
17	359	102	18	10	361	102	18	14	365	102	18	16	367	102	18
20	479	120	18	13	481	120	18	17	485	120	18	19	487	120	18
14	617	138	18	16	619	138	18	11	623	138	18	13	625	138	18
17	773	156	18	19	775	156	18	23	779	156	18	16	781	156	18
20	947	174	18	22	949	174	18	17	953	174	18	19	955	174	18
14	1139	192	18	7	1141	192	18	11	1145	192	18	13	1147	192	18
17	1349	210	18	10	1351	210	18	14	1355	210	18	16	1357	210	18
20	1577	228	18	22	1579	228	18	17	1583	228	18	19	1585	228	18
14	1823	246	18	16	1825	246	18	20	1829	246	18	13	1831	246	18
17	2087	264	18	19	2089	264	18	14	2093	264	18	16	2095	264	18
20	2369	282	18	13	2371	282	18	17	2375	282	18	19	2377	282	18
23	2669	300	18	16	2671	300	18	20	2675	300	18	22	2677	300	18
26	2987	318	18	28	2989	318	18	23	2993	318	18	25	2995	318	18
11	3323	336	18	13	3325	336	18	17	3329	336	18	10	3331	336	18
23	3677	354	18	25	3679	354	18	20	3683	354	18	22	3685	354	18
17	4049	372	18	10	4051	372	18	14	4055	372	18	16	4057	372	18
20	4439	390	18	13	4441	390	18	17	4445	390	18	19	4447	390	18
23	4847	408	18	25	4849	408	18	20	4853	408	18	22	4855	408	18
17	5273	426	18	19	5275	426	18	23	5279	426	18	16	5281	426	18
20	5717	444	18	22	5719	444	18	17	5723	444	18	19	5725	444	18
23	6179	462	18	16	6181	462	18	20	6185	462	18	22	6187	462	18

+ 4 → + 2 → + 4 → ....

+ 4 → + 2 → + 4 → ....

SD	B1	B1'	B1"	SD	B2	B2'	B2"	SD	B3	B3'	B3"	SD	B4	B4'	B4"
2	11			4	13			8	17			10	19		
11	47	36		13	49	36		8	53	36		10	55	36	
2	101	54	18	4	103	54	18	8	107	54	18	10	109	54	18
11	173	72	18	13	175	72	18	17	179	72	18	10	181	72	18
11	263	90	18	13	265	90	18	17	269	90	18	10	271	90	18
11	371	108	18	13	373	108	18	17	377	108	18	19	379	108	18
20	497	126	18	22	499	126	18	8	503	126	18	10	505	126	18
11	641	144	18	13	643	144	18	17	647	144	18	19	649	144	18
11	803	162	18	13	805	162	18	17	809	162	18	10	811	162	18
20	983	180	18	22	985	180	18	26	989	180	18	19	991	180	18
11	1181	198	18	13	1183	198	18	17	1187	198	18	19	1189	198	18
20	1397	216	18	22	1399	216	18	8	1403	216	18	10	1405	216	18
11	1631	234	18	13	1633	234	18	17	1637	234	18	19	1639	234	18
20	1883	252	18	22	1885	252	18	26	1889	252	18	19	1891	252	18
11	2153	270	18	13	2155	270	18	17	2159	270	18	10	2161	270	18
11	2441	288	18	13	2443	288	18	17	2447	288	18	19	2449	288	18
20	2747	306	18	22	2749	306	18	17	2753	306	18	19	2755	306	18
11	3071	324	18	13	3073	324	18	17	3077	324	18	19	3079	324	18
11	3413	342	18	13	3415	342	18	17	3419	342	18	10	3421	342	18
20	3773	360	18	22	3775	360	18	26	3779	360	18	19	3781	360	18
11	4151	378	18	13	4153	378	18	17	4157	378	18	19	4159	378	18
20	4547	396	18	22	4549	396	18	17	4553	396	18	19	4555	396	18
20	4961	414	18	22	4963	414	18	26	4967	414	18	28	4969	414	18
20	5393	432	18	22	5395	432	18	26	5399	432	18	10	5401	432	18
20	5843	450	18	22	5845	450	18	26	5849	450	18	19	5851	450	18
11	6311	468	18	13	6313	468	18	17	6317	468	18	19	6319	468	18
29	6797	486	18	31	6799	486	18	17	6803	486	18	19	6805	486	18

SD	B5	B5'	B5"	SD	B6	B6'	B6"	SD	B7	B7'	B7"	SD	B8	B8'	B8"
5	5			7	7			2	11			4	13		
5	23	18		7	25	18		11	29	18		4	31	18	
14	59	36	18	7	61	36	18	11	65	36	18	13	67	36	18
5	113	54	18	7	115	54	18	11	119	54	18	4	121	54	18
14	185	72	18	16	187	72	18	11	191	72	18	13	193	72	18
14	275	90	18	16	277	90	18	11	281	90	18	13	283	90	18
14	383	108	18	16	385	108	18	20	389	108	18	13	391	108	18
14	509	126	18	7	511	126	18	11	515	126	18	13	517	126	18
14	653	144	18	16	655	144	18	20	659	144	18	13	661	144	18
14	815	162	18	16	817	162	18	11	821	162	18	13	823	162	18
23	995	180	18	25	997	180	18	2	1001	180	18	4	1003	180	18
14	1193	198	18	16	1195	198	18	20	1199	198	18	4	1201	198	18
14	1409	216	18	7	1411	216	18	11	1415	216	18	13	1417	216	18
14	1643	234	18	16	1645	234	18	20	1649	234	18	13	1651	234	18
23	1895	252	18	25	1897	252	18	11	1901	252	18	13	1903	252	18
14	2165	270	18	16	2167	270	18	11	2171	270	18	13	2173	270	18
14	2453	288	18	16	2455	288	18	20	2459	288	18	13	2461	288	18
23	2759	306	18	16	2761	306	18	20	2765	306	18	22	2767	306	18
14	3083	324	18	16	3085	324	18	20	3089	324	18	13	3091	324	18
14	3425	342	18	16	3427	342	18	11	3431	342	18	13	3433	342	18
23	3785	360	18	25	3787	360	18	20	3791	360	18	22	3793	360	18
14	4163	378	18	16	4165	378	18	20	4169	378	18	13	4171	378	18
23	4559	396	18	16	4561	396	18	20	4565	396	18	22	4567	396	18
23	4973	414	18	25	4975	414	18	29	4979	414	18	22	4981	414	18
14	5405	432	18	16	5407	432	18	11	5411	432	18	13	5413	432	18
23	5855	450	18	25	5857	450	18	20	5861	450	18	22	5863	450	18
14	6323	468	18	16	6325	468	18	20	6329	468	18	13	6331	468	18

SD	B9	B9'	B9"	SD	B10	B10'	B10"	SD	B11	B11'	B11"	SD	B12	B12'	B12"
8	17			10	19			5	23			5	25		
8	35	18		10	37	18		5	41	18		7	43	18	
8	71	36	18	10	73	36	18	14	77	36	18	16	79	36	18
8	125	54	18	10	127	54	18	5	131	54	18	7	133	54	18
17	197	72	18	19	199	72	18	5	203	72	18	7	205	72	18
17	287	90	18	19	289	90	18	14	293	90	18	16	295	90	18
17	395	108	18	19	397	108	18	5	401	108	18	7	403	108	18
8	521	126	18	10	523	126	18	14	527	126	18	16	529	126	18
17	665	144	18	19	667	144	18	14	671	144	18	16	673	144	18
17	827	162	18	19	829	162	18	14	833	162	18	16	835	162	18
8	1007	180	18	10	1009	180	18	5	1013	180	18	7	1015	180	18
8	1205	198	18	10	1207	198	18	5	1211	198	18	7	1213	198	18
8	1421	216	18	10	1423	216	18	14	1427	216	18	16	1429	216	18
17	1655	234	18	19	1657	234	18	14	1661	234	18	16	1663	234	18
17	1907	252	18	19	1909	252	18	14	1913	252	18	16	1915	252	18
17	2177	270	18	19	2179	270	18	14	2183	270	18	16	2185	270	18
17	2465	288	18	19	2467	288	18	14	2471	288	18	16	2473	288	18
17	2771	306	18	19	2773	306	18	23	2777	306	18	25	2779	306	18
17	3095	324	18	19	3097	324	18	5	3101	324	18	7	3103	324	18
17	3437	342	18	19	3439	342	18	14	3443	342	18	16	3445	342	18
26	3797	360	18	28	3799	360	18	14	3803	360	18	16	3805	360	18
17	4175	378	18	19	4177	378	18	14	4181	378	18	16	4183	378	18
17	4571	396	18	19	4573	396	18	23	4577	396	18	25	4579	396	18
26	4985	414	18	28	4987	414	18	23	4991	414	18	25	4993	414	18
17	5417	432	18	19	5419	432	18	14	5423	432	18	16	5425	432	18
26	5867	450	18	28	5869	450	18	23	5873	450	18	25	5875	450	18
17	6335	468	18	19	6337	468	18	14	6341	468	18	16	6343	468	18

+ 4 → + 2 → + 4 → ....

+ 4 → + 2 → + 4 → ....

SD	C1	C1'	C1"	SD	C2	C2'	C2"	SD	C3	C3'	C3"	SD	C4	C4'	C4"
8	17			10	19			5	23			7	25		
16	59	42		7	61	42		13	65	42		13	67	42	
11	119	60	18	4	121	60	18	8	125	60	18	10	127	60	18
17	197	78	18	19	199	78	18	5	203	78	18	7	205	78	18
14	293	96	18	16	295	96	18	20	299	96	18	4	301	96	18
13	407	114	18	13	409	114	18	8	413	114	18	10	415	114	18
17	539	132	18	10	541	132	18	14	545	132	18	16	547	132	18
23	689	150	18	16	691	150	18	20	695	150	18	22	697	150	18
20	857	168	18	22	859	168	18	17	863	168	18	19	865	168	18
8	1043	186	18	10	1045	186	18	14	1049	186	18	7	1051	186	18
14	1247	204	18	16	1249	204	18	11	1253	204	18	13	1255	204	18
20	1469	222	18	13	1471	222	18	17	1475	222	18	19	1477	222	18
17	1709	240	18	10	1711	240	18	14	1715	240	18	16	1717	240	18
23	1967	258	18	25	1969	258	18	20	1973	258	18	22	1975	258	18
11	2243	276	18	13	2245	276	18	17	2249	276	18	10	2251	276	18
17	2537	294	18	19	2539	294	18	14	2543	294	18	16	2545	294	18
23	2849	312	18	16	2851	312	18	20	2855	312	18	22	2857	312	18
20	3179	330	18	13	3181	330	18	17	3185	330	18	19	3187	330	18
17	3527	348	18	19	3529	348	18	14	3533	348	18	16	3535	348	18
23	3893	366	18	25	3895	366	18	29	3899	366	18	13	3901	366	18
20	4277	384	18	22	4279	384	18	17	4283	384	18	19	4285	384	18
26	4679	402	18	19	4681	402	18	23	4685	402	18	25	4687	402	18
23	5099	420	18	7	5101	420	18	11	5105	420	18	13	5107	420	18
20	5537	438	18	22	5539	438	18	17	5543	438	18	19	5545	438	18
26	5993	456	18	28	5995	456	18	32	5999	456	18	7	6001	456	18
23	6467	474	18	25	6469	474	18	20	6473	474	18	22	6475	474	18
29	6959	492	18	22	6961	492	18	26	6965	492	18	28	6967	492	18

SD	C5	C5'	C5"	SD	C6	C6'	C6"	SD	C7	C7'	C7"	SD	C8	C8'	C8"
5	5			7	7			2	11			4	13		
11	29	24		4	31	24		8	35	24		10	37	24	
8	71	42	18	10	73	42	18	14	77	42	18	16	79	42	18
5	131	60	18	7	133	60	18	11	137	60	18	13	139	60	18
11	209	78	18	4	211	78	18	8	215	78	18	10	217	78	18
8	305	96	18	10	307	96	18	5	311	96	18	7	313	96	18
14	419	114	18	7	421	114	18	11	425	114	18	13	427	114	18
11	551	132	18	13	553	132	18	17	557	132	18	19	559	132	18
8	701	150	18	10	703	150	18	14	707	150	18	16	709	150	18
23	869	168	18	16	871	168	18	20	875	168	18	22	877	168	18
11	1055	186	18	13	1057	186	18	8	1061	186	18	10	1063	186	18
17	1259	204	18	10	1261	204	18	14	1265	204	18	16	1267	204	18
14	1481	222	18	16	1483	222	18	20	1487	222	18	22	1489	222	18
11	1721	240	18	13	1723	240	18	17	1727	240	18	19	1729	240	18
26	1979	258	18	19	1981	258	18	23	1985	258	18	25	1987	258	18
14	2255	276	18	16	2257	276	18	11	2261	276	18	13	2263	276	18
20	2549	294	18	13	2551	294	18	17	2555	294	18	19	2557	294	18
17	2861	312	18	19	2863	312	18	23	2867	312	18	25	2869	312	18
14	3191	330	18	16	3193	330	18	20	3197	330	18	22	3199	330	18
20	3539	348	18	13	3541	348	18	17	3545	348	18	19	3547	348	18
17	3905	366	18	19	3907	366	18	14	3911	366	18	16	3913	366	18
23	4289	384	18	16	4291	384	18	20	4295	384	18	22	4297	384	18
20	4691	402	18	22	4693	402	18	26	4697	402	18	28	4699	402	18
8	5111	420	18	10	5113	420	18	14	5117	420	18	16	5119	420	18
23	5549	438	18	16	5551	438	18	20	5555	438	18	22	5557	438	18
11	6005	456	18	13	6007	456	18	8	6011	456	18	10	6013	456	18
26	6479	474	18	19	6481	474	18	23	6485	474	18	25	6487	474	18

SD	C9	C9'	C9"	SD	C10	C10'	C10"	SD	C11	C11'	C11"	SD	C12	C12'	C12"
8	17			10	19			5	23			7	25		
5	41	24		7	43	24		11	47	24		13	49	24	
11	83	42	18	13	85	42	18	17	89	42	18	10	91	42	18
8	143	60	18	10	145	60	18	14	149	60	18	7	151	60	18
5	221	78	18	7	223	78	18	11	227	78	18	13	229	78	18
11	317	96	18	13	319	96	18	8	323	96	18	10	325	96	18
8	431	114	18	10	433	114	18	14	437	114	18	16	439	114	18
14	563	132	18	16	565	132	18	20	569	132	18	13	571	132	18
11	713	150	18	13	715	150	18	17	719	150	18	10	721	150	18
17	881	168	18	19	883	168	18	23	887	168	18	25	889	168	18
14	1067	186	18	16	1069	186	18	11	1073	186	18	13	1075	186	18
11	1271	204	18	13	1273	204	18	17	1277	204	18	19	1279	204	18
17	1493	222	18	19	1495	222	18	23	1499	222	18	7	1501	222	18
14	1733	240	18	16	1735	240	18	20	1739	240	18	13	1741	240	18
20	1991	258	18	22	1993	258	18	26	1997	258	18	28	1999	258	18
17	2267	276	18	19	2269	276	18	14	2273	276	18	16	2275	276	18
14	2561	294	18	16	2563	294	18	20	2567	294	18	22	2569	294	18
20	2873	312	18	22	2875	312	18	26	2879	312	18	19	2881	312	18
8	3203	330	18	10	3205	330	18	14	3209	330	18	7	3211	330	18
14	3551	348	18	16	3553	348	18	20	3557	348	18	22	3559	348	18
20	3917	366	18	22	3919	366	18	17	3923	366	18	19	3925	366	18
8	4301	384	18	10	4303	384	18	14	4307	384	18	16	4309	384	18
14	4703	402	18	16	4705	402	18	20	4709	402	18	13	4711	402	18
11	5123	420	18	13	5125	420	18	17	5129	420	18	10	5131	420	18
17	5561	438	18	19	5563	438	18	23	5567	438	18	25	5569	438	18
14	6017	456	18	16	6019	456	18	11	6023	456	18	13	6025	456	18
20	6491	474	18	22	6493	474	18	26	6497	474	18	28	6499	474	18

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+ 4 → + 2 → + 4 → ....

Table 6-A2 : Quadratic Polynomials of the Spiral-Graphs belonging to the "Prime Number Spiral Systems" **P18-A, P18-B** and **P18-C** (with the **2. Differential = 18**)

Spiral Graph System	Spiral Graph	Number Sequence of Spiral Graph	Quadratic Polynomial 1 (calculated with the first 3 numbers of the given sequence)	Quadratic Polynomial 2 (calculated with 3 numbers starting with the **2.** Number of the sequence)	Quadratic Polynomial 3 (calculated with 3 numbers starting with the **3.** Number of the sequence)	Quadratic Polynomial 4 (calculated with 3 numbers starting with the **4.** Number of the sequence)
P18-A	A1	5 , 35 , 83 , 149 , 233 , 335 ,......	$f_1 (x) = 9 x^2 + 3 x - 7$	$f_2 (x) = 9 x^2 + 21 x + 5$	$f_3 (x) = 9 x^2 + 39 x + 35$	$f_4 (x) = 9 x^2 + 57 x + 83$
	A2	7 , 37 , 85 , 151 , 235 , 337 ,......	$f_1 (x) = 9 x^2 + 3 x - 5$	$f_2 (x) = 9 x^2 + 21 x + 7$	$f_3 (x) = 9 x^2 + 39 x + 37$	$f_4 (x) = 9 x^2 + 57 x + 85$
	A3	11 , 41 , 89 , 155 , 239 , 341 ,......	$f_1 (x) = 9 x^2 + 3 x - 1$	$f_2 (x) = 9 x^2 + 21 x + 11$	$f_3 (x) = 9 x^2 + 39 x + 41$	$f_4 (x) = 9 x^2 + 57 x + 89$
	A4	1 , 13 , 43 , 91 , 157 , 241 ,......	$f_1 (x) = 9 x^2 - 15 x + 7$	$f_2 (x) = 9 x^2 + 3 x + 1$	$f_3 (x) = 9 x^2 + 21 x + 13$	$f_4 (x) = 9 x^2 + 39 x + 43$
	A5	5 , 17 , 47 , 95 , 161 , 245 ,......	$f_1 (x) = 9 x^2 - 15 x + 11$	$f_2 (x) = 9 x^2 + 3 x + 5$	$f_3 (x) = 9 x^2 + 21 x + 17$	$f_4 (x) = 9 x^2 + 39 x + 47$
	A6	7 , 19 , 49 , 97 , 163 , 247 ,......	$f_1 (x) = 9 x^2 - 15 x + 13$	$f_2 (x) = 9 x^2 + 3 x + 7$	$f_3 (x) = 9 x^2 + 21 x + 19$	$f_4 (x) = 9 x^2 + 39 x + 49$
	A7	11 , 23 , 53 , 101 , 167 , 251 ,......	$f_1 (x) = 9 x^2 - 15 x + 17$	$f_2 (x) = 9 x^2 + 3 x + 11$	$f_3 (x) = 9 x^2 + 21 x + 23$	$f_4 (x) = 9 x^2 + 39 x + 53$
	A8	13 , 25 , 55 , 103 , 169 , 253 ,......	$f_1 (x) = 9 x^2 - 15 x + 19$	$f_2 (x) = 9 x^2 + 3 x + 13$	$f_3 (x) = 9 x^2 + 21 x + 25$	$f_4 (x) = 9 x^2 + 39 x + 55$
	A9	17 , 29 , 59 , 107 , 173 , 257 ,......	$f_1 (x) = 9 x^2 - 15 x + 23$	$f_2 (x) = 9 x^2 + 3 x + 17$	$f_3 (x) = 9 x^2 + 21 x + 29$	$f_4 (x) = 9 x^2 + 39 x + 59$
	A10	19 , 31 , 61 , 109 , 175 , 259 ,......	$f_1 (x) = 9 x^2 - 15 x + 25$	$f_2 (x) = 9 x^2 + 3 x + 19$	$f_3 (x) = 9 x^2 + 21 x + 31$	$f_4 (x) = 9 x^2 + 39 x + 61$
	A11	23 , 35 , 65 , 113 , 179 , 263 ,......	$f_1 (x) = 9 x^2 - 15 x + 29$	$f_2 (x) = 9 x^2 + 3 x + 23$	$f_3 (x) = 9 x^2 + 21 x + 35$	$f_4 (x) = 9 x^2 + 39 x + 65$
	A12	25 , 37 , 67 , 115 , 181 , 265 ,......	$f_1 (x) = 9 x^2 - 15 x + 31$	$f_2 (x) = 9 x^2 + 3 x + 25$	$f_3 (x) = 9 x^2 + 21 x + 37$	$f_4 (x) = 9 x^2 + 39 x + 67$
P18-B	B1	11 , 47 , 101 , 173 , 263 , 371 ,......	$f_1 (x) = 9 x^2 + 9 x - 7$	$f_2 (x) = 9 x^2 + 27 x + 11$	$f_3 (x) = 9 x^2 + 45 x + 47$	$f_4 (x) = 9 x^2 + 63 x + 101$
	B2	13 , 49 , 103 , 175 , 265 , 373 ,......	$f_1 (x) = 9 x^2 + 9 x - 5$	$f_2 (x) = 9 x^2 + 27 x + 13$	$f_3 (x) = 9 x^2 + 45 x + 49$	$f_4 (x) = 9 x^2 + 63 x + 103$
	B3	17 , 53 , 107 , 179 , 269 , 377 ,......	$f_1 (x) = 9 x^2 + 9 x - 1$	$f_2 (x) = 9 x^2 + 27 x + 17$	$f_3 (x) = 9 x^2 + 45 x + 53$	$f_4 (x) = 9 x^2 + 63 x + 107$
	B4	1 , 19 , 55 , 109 , 181 , 271 ,......	$f_1 (x) = 9 x^2 - 9 x + 1$	$f_2 (x) = 9 x^2 + 9 x + 1$	$f_3 (x) = 9 x^2 + 27 x + 19$	$f_4 (x) = 9 x^2 + 45 x + 55$
	B5	5 , 23 , 59 , 113 , 185 , 275 ,......	$f_1 (x) = 9 x^2 - 9 x + 5$	$f_2 (x) = 9 x^2 + 9 x + 5$	$f_3 (x) = 9 x^2 + 27 x + 23$	$f_4 (x) = 9 x^2 + 45 x + 59$
	B6	7 , 25 , 61 , 115 , 187 , 277 ,......	$f_1 (x) = 9 x^2 - 9 x + 7$	$f_2 (x) = 9 x^2 + 9 x + 7$	$f_3 (x) = 9 x^2 + 27 x + 25$	$f_4 (x) = 9 x^2 + 45 x + 61$
	B7	11 , 29 , 65 , 119 , 191 , 281 ,......	$f_1 (x) = 9 x^2 - 9 x + 11$	$f_2 (x) = 9 x^2 + 9 x + 11$	$f_3 (x) = 9 x^2 + 27 x + 29$	$f_4 (x) = 9 x^2 + 45 x + 65$
	B8	13 , 31 , 67 , 121 , 193 , 283 ,......	$f_1 (x) = 9 x^2 - 9 x + 13$	$f_2 (x) = 9 x^2 + 9 x + 13$	$f_3 (x) = 9 x^2 + 27 x + 31$	$f_4 (x) = 9 x^2 + 45 x + 67$
	B9	17 , 35 , 71 , 125 , 197 , 287 ,......	$f_1 (x) = 9 x^2 - 9 x + 17$	$f_2 (x) = 9 x^2 + 9 x + 17$	$f_3 (x) = 9 x^2 + 27 x + 35$	$f_4 (x) = 9 x^2 + 45 x + 71$
	B10	19 , 37 , 73 , 127 , 199 , 289 ,......	$f_1 (x) = 9 x^2 - 9 x + 19$	$f_2 (x) = 9 x^2 + 9 x + 19$	$f_3 (x) = 9 x^2 + 27 x + 37$	$f_4 (x) = 9 x^2 + 45 x + 73$
	B11	23 , 41 , 77 , 131 , 203 , 293 ,......	$f_1 (x) = 9 x^2 - 9 x + 23$	$f_2 (x) = 9 x^2 + 9 x + 23$	$f_3 (x) = 9 x^2 + 27 x + 41$	$f_4 (x) = 9 x^2 + 45 x + 77$
	B12	25 , 43 , 79 , 133 , 205 , 295 ,......	$f_1 (x) = 9 x^2 - 9 x + 25$	$f_2 (x) = 9 x^2 + 9 x + 25$	$f_3 (x) = 9 x^2 + 27 x + 43$	$f_4 (x) = 9 x^2 + 45 x + 79$
P18-C	C1	17 , 59 , 119 , 197 , 293 , 407 ,......	$f_1 (x) = 9 x^2 + 15 x - 7$	$f_2 (x) = 9 x^2 + 33 x + 17$	$f_3 (x) = 9 x^2 + 51 x + 59$	$f_4 (x) = 9 x^2 + 69 x + 119$
	C2	19 , 61 , 121 , 199 , 295 , 409 ,......	$f_1 (x) = 9 x^2 + 15 x - 5$	$f_2 (x) = 9 x^2 + 33 x + 19$	$f_3 (x) = 9 x^2 + 51 x + 61$	$f_4 (x) = 9 x^2 + 69 x + 121$
	C3	23 , 65 , 125 , 203 , 299 , 413 ,......	$f_1 (x) = 9 x^2 + 15 x - 1$	$f_2 (x) = 9 x^2 + 33 x + 23$	$f_3 (x) = 9 x^2 + 51 x + 65$	$f_4 (x) = 9 x^2 + 69 x + 125$
	C4	1 , 25 , 67 , 127 , 205 , 301 ,......	$f_1 (x) = 9 x^2 - 3 x - 5$	$f_2 (x) = 9 x^2 + 15 x + 1$	$f_3 (x) = 9 x^2 + 33 x + 25$	$f_4 (x) = 9 x^2 + 51 x + 67$
	C5	5 , 29 , 71 , 131 , 209 , 305 ,......	$f_1 (x) = 9 x^2 - 3 x - 1$	$f_2 (x) = 9 x^2 + 15 x + 5$	$f_3 (x) = 9 x^2 + 33 x + 29$	$f_4 (x) = 9 x^2 + 51 x + 71$
	C6	1 , 7 , 31 , 73 , 133 , 211 ,......	$f_1 (x) = 9 x^2 - 21 x + 13$	$f_2 (x) = 9 x^2 - 3 x + 1$	$f_3 (x) = 9 x^2 + 15 x + 7$	$f_4 (x) = 9 x^2 + 33 x + 31$
	C7	5 , 11 , 35 , 77 , 137 , 215 ,......	$f_1 (x) = 9 x^2 - 21 x + 17$	$f_2 (x) = 9 x^2 - 3 x + 5$	$f_3 (x) = 9 x^2 + 15 x + 11$	$f_4 (x) = 9 x^2 + 33 x + 35$
	C8	7 , 13 , 37 , 79 , 139 , 217 ,......	$f_1 (x) = 9 x^2 - 21 x + 19$	$f_2 (x) = 9 x^2 - 3 x + 7$	$f_3 (x) = 9 x^2 + 15 x + 13$	$f_4 (x) = 9 x^2 + 33 x + 37$
	C9	11 , 17 , 41 , 83 , 143 , 221 ,......	$f_1 (x) = 9 x^2 - 21 x + 23$	$f_2 (x) = 9 x^2 - 3 x + 11$	$f_3 (x) = 9 x^2 + 15 x + 17$	$f_4 (x) = 9 x^2 + 33 x + 41$
	C10	13 , 19 , 43 , 85 , 145 , 223 ,......	$f_1 (x) = 9 x^2 - 21 x + 25$	$f_2 (x) = 9 x^2 - 3 x + 13$	$f_3 (x) = 9 x^2 + 15 x + 19$	$f_4 (x) = 9 x^2 + 33 x + 43$
	C11	17 , 23 , 47 , 89 , 149 , 227 ,......	$f_1 (x) = 9 x^2 - 21 x + 29$	$f_2 (x) = 9 x^2 - 3 x + 17$	$f_3 (x) = 9 x^2 + 15 x + 23$	$f_4 (x) = 9 x^2 + 33 x + 47$
	C12	19 , 25 , 49 , 91 , 151 , 229 ,......	$f_1 (x) = 9 x^2 - 21 x + 31$	$f_2 (x) = 9 x^2 - 3 x + 19$	$f_3 (x) = 9 x^2 + 15 x + 25$	$f_4 (x) = 9 x^2 + 33 x + 49$

5 The share of Prime Numbers in the analysed number sequences

To find out how high the share of prime numbers really is, in the prime-number-sequences shown in Table 6-A1to 6-C1, I have carried out a random "spot check" in a few of these sequences.

For these analysis I chose the three Prime Number Spiral Graphs (-sequences) **B3** , **Q3** and **P20-G1** (see FIG 6-A to 6-C and Table 6-A1to 6-C1) , because of their high share in prime numbers at the beginning of the sequence.

With the help of an excel-table I then extended these three sequences up to numbers >2,500,000,000 → **see Table 1**

Then I picked out a longer section at the beginning and four sections out of the further run of these extended sequences, to analyse them in regards to their share in Prime Numbers.

For the other four sections I picked sections of 8 numbers out of the number areas : 2,500,000 ; 25,000,000 ; 250,000,000 and 2,500,000,000 for each of the three number-sequences.

I then marked the Prime Numbers in each sequence with yellow color. I also marked numbers which are not divisible by 2, 3 or 5 with red color, in the different sections of these sequences.

By looking over Table 1 it is obvious that the share in prime numbers is dropping after the beginning of the sequences, which has a share in prime numbers of around 70 – 75 %.

But it seems that the share in prime number is striving for around 25 – 35 % in the long run. The distribution of prime numbers in the number area 0 – 2,500,000,000 seems to be relatively evenly.

However there are also sections where only few prime numbers occur (e.g. in the second sections in sequence Q3 and P20-G1, in the number area 2,500,000)

In any case, it would be worth to further extend the three prime-numbers sequences B3, Q3 and P20-G1 shown in Table 1, and a few other sequences which also have high shares in prime numbers (e.g. the number-sequences A9, C2, C5, P20-G3, P20-H1, S1 etc. → see Table 6-A1 to 6-C1), to analyse them for their content of prime numbers. This could be done with special software which automatically extends and analyses these sequences for prime numbers.

6 To the periodic occurrence of prime factors in non-prime-numbers

When I set-up **Table 1**, I noticed that some of the prime factors of the non-prime-numbers occured at equal intervals in the sequences B3, Q3 and P20-G1. It also appeared that only defined prime factors occur in every sequence.
I have marked some of these prime factors in red color in Table 1(→ see column "prime-factors"). For example the prime factors 17 and 23 in sequence B3 and the prime factors 23 and 71 in sequence P20-G1occur at equal intervals.

These peculiarities of some of the prime factors shown in Table 1, forced me to do an analysis in regards to a possible "periodic behavior" of the prime factors which form the non-prime-numbers in the analysed prime number spiral graphs .

Besides the three prime-number-sequences B3, Q3 and P20-G1, I further chose the sequences K5, S1 and B33 for this analysis.
The two sequences S1 and B33 where chosen because they have the same number-endings-sequences and the same "sums of the digits"-sequences as the number sequences Q3 and B3 (→ see Table 2 and 3).
Therefore it was interesting to see what effects these similarities have on the periodic distribution of the prime-factors in these sequences.

Here the sequence B33 was developed, by a further extension of Table 6-A1on the righthand side , by adding alternately the numbers 2 or 4.

The number-sequence K5 was chosen, because it contains many small prime-factors at the beginning of the sequence. (This property was used to explain the origin-principle of the prime-number-sequences shown in FIG 6-A to 6-C in a graphic way (in FIG. 7). I will come back to this point later !)

The results of this analysis regarding the periodic behavior of some small prime-factors are shown in Table 4 , Table 5-A and Table 5-B. (→ see next pages !)

These tables give a good insight into the periodic behavior of the smallest prime-factors contained in the non-prime-numbers of the chosen sequences.

The following list describes the most remarkable properties of these periodic occurring prime-factors : (→ see Table 4, 5-A and 5-B)

1.) - All prime-factors, which form the non-prime-numbers in the analysed number-sequences, recur in defined periods. And this principle seems to apply to all prime-number-sequences derived from the Spiral-Graphs shown in FIG 6-A to FIG 6-C.

2.) - The following general rule applies for these periods :
The smaller the prime-factor, the smaller is the period in which the prime-factor occurs in each number sequence.

3.) - Further the following remarkable rule applies :

The period-length of every prime-factor, expressed in spacings (lines in the table), or the sum of the period-lengths (if there are 2 different ones !), is identical to the value of the prime-factor !

For example : Prime Factor 13 in number-sequence " B3 " occurs periodically in every 13th line of the Table 4.
Or Prime Factor 23 occurs with the two alternating period-lengths 2 and 21, which add up to 23, which again is exactly equal to the prime factor itself ! etc.

It is notable that the number of spacings between two numbers (→ lines in the table between two numbers) corresponds with the same number of "winds" of the Square-Root-Spiral , which lie between these two numbers !

4.) - By comparing the prime factors, which occur in the non-prime numbers of the analysed number-sequences, it is notable that only specific smallest prime-factors occur in each number sequence (→ see columns " prime-factors of non-prime-numbers" in Table 4, 5-A and 5-B)

For example these are the following prime-factors :

In number-sequence	B3 :	13, 17, 23, 29, 43, 53, 61,…
" " "	**Q3 :**	**11, 13, 31, 37, 73, 89,….**
" " "	P20-G1 :	13, 23, 31, 67, 71, 73,….
" " "	K5 :	7, 11, 13, 17, 29, 37,….
" " "	**S1 :**	**11, 13, 31, 37, 73, 89,….**
" " "	B33 :	11, 13, 29, 43, 53, 59, 61….

This is a clear indication for a "blueprint" which controls the composition of the non-prime-numbers in each sequence. In each number-sequence specific smallest prime-factors are completely absent. This leads to the following conclusion :

" The none-prime-numbers in each number-sequence (shown in Table 6-A1 to 6-C1) are formed by a defined number of specific prime-factors, which recur in defined periods ! "

5.) - A comparison of the prime- factor distribution in the number-sequences **Q3** and **S1** clearly shows, that the non-prime-numbers in these two sequences **consist of exactly the same prime-factors !! Even the periods** in which these prime factors occur **are exactly the same !!** The only difference between the non-prime numbers in these two number-sequences is that the non-prime numbers are made of different combinations of prime-factors and that they are distributed in a different way in these two number-sequence.

However this comparison shows that number-sequences of **different spiral-graph-systems**, which have the same number endings sequence and the same "sums of the digits sequence" (see Table 2 and 3), seem to contain exactly the same prime-factors with the same periods in their non-prime-numbers !!!

But a comparison of the two number-sequences B3 and B33, which belong to the **same spiral-graph system** P18-B (see FIG 6-A), and which also have the same number endings sequences and the same sums of the digits sequences do not contain exactly the same prime-factors with the same periods !

As mentioned on the last page, I have developed the number-sequence B33 through a further extension of Table 6-A1on the righthand side , by adding alternately the numbers 2 or 4.
I did this to have another pair of sequences (B3 and B33) which according to Table 2 and 3 also have the same number endings sequences and the same sums of the digits sequences".

But comparing the prime-factors, which occur in Table 4 and 5-B in the non-prime-numbers of the two sequences B3 and B33, doesn't show the same matching of prime-factors and periods as in the sequences Q3 and S1.
The most prime factors in the sequences B3 and B33 may be the same, but the periods in which they occur are clearly different. And it is also noticeable that both sequences also contain a few different prime-factors.

7 Graphic explanation of the origin of the periodic occuring prime factors

Now I want to show a "graphic explanation" for the origin of the periodic occurring prime factors described in Table 4, 5-A and 5-B.

For this please have a look at → **FIG 7**

Here the spiral-graph **K5** is drawn in **red** color. (→ see also FIG 6-C)
I have chosen the spiral-graph K5 because it contains many non-prime-numbers at the start.
Because of that it can be demonstrated, that the three non-prime-numbers **49, 187** and **289** are formed by "points of intersection" of the three "number-group-spiral-systems" which contain either numbers divisible by **7, 11** or **17**.

→ (**Ref.:** As already mentioned in the abstract and in the introduction, all natural numbers divisible by the same prime number lie in defined **"number-group-spiral-systems"**. To get a better understanding of this property, please have a read through my introduction to the Square Root Spiral :

→ " **The ordered distribution of natural numbers on the Square Root Spiral "**

In this study the number-group-spiral-systems, which contain the numbers divisible by 7, 11 and 17 are shown, and the general rule which defines these spiral-systems is described → see **chapter 5.2** in the mentioned study !)

For clearness I have only shown one "spiral-graph-system" of each number-group-spiral-system in FIG 7 And from each of these systems I have only shown 3 to 4 spiralarms !

Besides the spiral-graph K5, the number-group-spiral-system P1 is shown, which contains numbers divisible by 17 (blue), as well as the system N2 (orange), which contains numbers divisible by 7 , and the system N2 (pink), which contains numbers divisible by 11 (see FIG 7).
Now it is easy to see in FIG 7, that the course (curvature) of the spiral-graph K5 is already fully defined by periodic points of intersection of the three mentioned number-group-spiral-systems (which contain the numbers divisible by 7, 11 and 17) with the square root spiral.

Other defined number-group-spiral-systems also contribute with periodic points of intersection to the formation of non-prime-numbers on spiral-graph K5, but the course of spiral-graph 5 is already defined by the mentioned spiral-systems.

The **Appendix** shows a diagram of the prime-number-spiral-graph B3 with the specification of the exact polar coordinates of the points of intersection of this graph with the square-root-spiral (→ positions of the natural numbers which lie on this graph). These coordinates might be helpful for an exact analysis of this spiral-graph !

8 Final Comment

Every prime-number-spiral-graph presented in FIG 6-A to 6-C, shows periodicities in the distribution of the prime factors of it's non-prime numbers ! Tables 4, 5-A and 5-B are a first proof for this proposition.

And the share of Prime Numbers as well as the distribution of Prime Numbers on a certain prime-number-spiral-graph is a result of the periodic occurring prime factors which form the non-prime numbers in this graph !

The distribution of the periodic occuring prime factors is defined by the number-group-spiral-systems which I have described in my introduction study to the Square Root Spiral. (→ see arXiv – Archiv) The Title of this study is :

" The ordered distribution of natural numbers on the Square Root Spiral " [1]

The general rule, which defines the arrangement of the mentioned number-group-spiral-systems on the Square Root Spiral, can be found in **chapter 5.2** in the above mentioned study ! (→ A reference to these number-group-spiral-systems is also given on the previous page ! → see righthand side).

Similar to the prime-number-spiral-graphs in this study , the spiralarms of the mentioned number-group-spiral-systems are also clear defined by quadratic polynomials.

The periodic occuring prime factors in the non-prime numbers of the prime-number-spiral-graphs can be explained by periodic points of intersection of certain spiralarms of the number-group-spiral-systems with the Square Root Spiral (→ see example in FIG 7)

In this connection I also want to refer to my 3. Study which I intend to file with the arXiv – Archiv. The title of this study is :

→ " The logic of the prime number distribution" [3]

In this study the general distribution of the prime numbers is decribed with a simple "Wave Model" in a visual way.
The base of this "Wave Model" is the fact, that two Prime Number Sequences (SQ1 & SQ2), which seem to contain all prime numbers, recur in themselves over and over again with increasing wave-lengths, in a very similar way as "Undertones" derive from a defined fundamental frequency **f**.

Undertones are the inversion of Overtones, which are known by every musician. Overtones (harmonics) are integer multiples of a fundamental frequency **f**.

The continuous recurrence of these number-sequences (SQ1 + SQ2) in themselves can be considered as the principle of creation of the non-prime numbers in these two number sequences. Non-prime-numbers are created on places in these number sequences where there is interference caused by the recurrences of these number-sequences. On the other hand prime numbers represent places in these number-sequences (SQ1 + SQ2) where there is no interference caused through the recurrences of this number-sequences.

The logic of this "Wave Model" is really easy to understand !
→ Please have a look at Table 2 in the mentioned paper !

The two Prime Number Sequences SQ1 & SQ2 which I mentioned, are actually easy to see in FIG 6-A. Following the winds of the square root spiral it is easy to see, that all prime numbers lie on two sequences of numbers, which are shifted to each other by two numbers, and where the difference between two successive numbers in each sequence is always 6.
By the way,... the "Prime Number Spiral Graphs" shown in FIG 6-A contain the same numbers as the mentioned Number Sequences SQ1 & SQ2 ! These are the natural numbers which are not divisible by 2 and 3.

The "periodic phenomenons" described in this work and in the other mentioned study, which are responsible for the distribution of the non-prime-numbers and prime numbers, should definitely be further analysed in more detail !

From the point of view of the Square Root Spiral, the distribution of the prime numbers seems to be clearly ordered. However this ordering principle is hard to grasp, because it is defined by the spatial complex interference of the mentioned number-group-spiral-systems , which is rapidly increasing from the centre of the square root spiral towards infinity.

In this connection I want to refer to another study, which I intend to file with the ArXiv-archive, which shows surprising similarities between the periodic behavior of the prime factors in the non-prime-numbers of the spiral-graphs shown in FIG 6-A to 6-C and the periodic behavior of prime factors which occur in Fibonacci-Numbers !

The title of this study is :

→ **" The mathematical origin of natural Fibonacci-Sequences, and the periodic distribution of prime factors in these sequences." [4]**

Because my graphical analysis of the Square-Root-Spiral seems to open up new territory, I can't really give many references for my work.

However I found an interesting webside, which was set up by Mr. Robert Sachs.

And this webside deals with a special "Number Spiral", which is closely related to the well known Ulam-Spiral and to the Square Root Spiral as well. And on this "Number-Spiral" prime numbers also accumulate on defined graphs in a very similar fashion than shown in my study in FIG 6-A to 6-C.

The Number Spiral, analysed by Mr. Sachs, is approximately winded three times tighter in comparison with the Square Root Spiral, in a way that the three square-number spiralarms Q1, Q2 and Q3 of the Square Root Spiral (see FIG. 1) are congruent on one straight line !
Therefore a comparison of my analysis results with the results of the study from Mr. Robert Sachs could be very helpful for further discoveries !

Mr. Sachs gave me permission to show some sections of his analysis in my paper.

From the 10 chapters shown on the webside of Mr. Sachs I want to show the chapters which have a close connection to my findings. These are as follows :

1.) – Introduction ; 2.) – Product Curves ; 3.) – Offset Curves ; 5.) – Quadratic Polynomials ; 7.) – Prime Numbers ; 9.) - Formulas

Please have a look to **chapter 9** with the title **" The Number Spiral"** on **page 28** which shows the above mentioned chapters.

Images from the analysis of Mr. Sachs are named as follows: FIG. NS-1 to NS-18

The webside of Mr. Sachs, which deals with the mentioned Number Spiral can be found under the following weblink : → www.numberspiral.com

Following **chapter 9,** I then compared the Square Root Spiral with the Number Spiral and the Ulam Spiral, in regards to the arrangement of some selected "Reference Graphs". This comparison is shown in **chapter 10** → see **page 36**

I consider this comparison only as a first little step of a much more extensive analysis, which should be carried out here, in regards to the arrangement of prime numbers and non-prime-numbers on spirals with different spiral intensities, where the square numbers are located on a defined number of graphs.

In the following I want to show a priority list of some discoveries shown in my studies 1 to 4, where further mathematical analyses should be done, to explain this findings !

Special attention should here be paid to the distribution of <u>prime factors</u> in the <u>non-prime numbers</u> of the analysed spiral graphs and number sequences ! :

Priority List of discoveries, suggested for further mathematical analysis :

		Study
1	- General rule which defines the Number-Group-Spiral-Systems → chapter 5.2	- [1]
2	- Prime Number Spiral Systems shown in FIG 6-A to 6-C → chapter 3 – <u>in this stuc</u>	- [2]
3	- Periodic occurrence of the prime factors in the non-prime-numbers of the Prime Number Spiral Graphs & their period lengths (→ Chapt. 6 /Tab. 4, 5-A, 5-	- [2]
4	- The meaning of the "Sums of the digits – Sequences" and "Number-endings Sequences" in the whole context (→ described in Chapter 4.3 and 4.4)	- [2]
5	- Why do the number-endings-sequences and "sums of the digits sequences" usually cover **5** successive numbers of the analysed sequences ? (→ 4.3/4.4)	- [2]
6	- The interlaced occurrence of the special Prime Number Sequences SQ1 + SQ on the Square Root Spiral → (see FIG 8) **- [2]** ; → also see my 3. Study !	- [3]
7	- Pronics-Graphs and Product Curves on the Square Root Spiral & Number Spir	- [2]
8	- The periodic occurrence of prime factors in the natural Fibonacci-Sequence	- [4]
9	- The distribution of the Square Numbers on 3 highly symmetrical spiral graphs	- [1]
10	- Interval π between winds of the Square Root Spiral for sqrt → ∞ (Chapt. 1 &	- [2]
11	- Difference-Graph $f(x) = 2 (5x^2 - 7x + 3)$ shown in FIG. 16 / in the Appendix	- [1]

In December 2005 and June 2006 I sent the most findings shown in this study here to some universities in Germany for an assessment. But unfortunately there wasn't much response ! That's why I decided to publish my discoveries here !

Prof. S.J. Patterson from the University of Goettingen found some of my discoveries very interesting. He was especially interested in the spiral graphs which contain the Prime Numbers (shown in FIG 6-A to 6-C).
These spiral graphs are special quadratic polynomials, which are of great interest to Prime Number Theory. For example the quadratic polynomial B3 in FIG 6-A → $B3 = F(x) = 9x^2 + 27x + 17$ (or $9x^2 + 9x – 1$)
Prof. Ernst Wilhelm Zink from the Humboldt-University in Berlin also found my study very interesting and he organized a mathematical analysis of the spiral-graphs shown in FIG 6-A to FIG 6-C.

This mathematical analysis was carried out by Mr. Kay Schoenberger, a student of mathematics on the Humboldt-University of Berlin, who was working on his dissertation. The results of this mathematical analysis is shown in my first study :

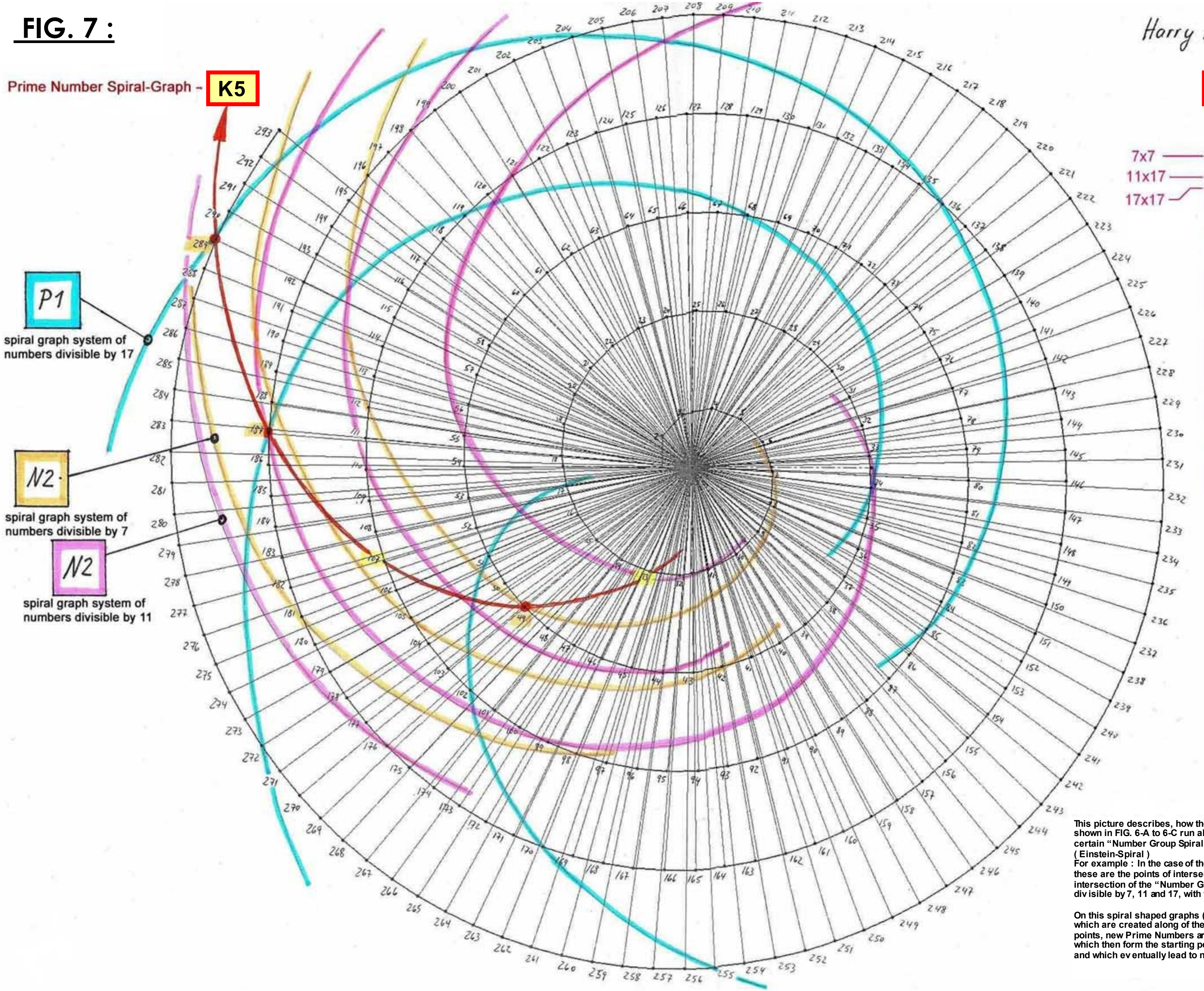
→ " **The ordered distribution of natural numbers on the Square Root Spiral " [1]**

The Square Root Spiral and the Number Spiral are interesting tools to find (spatial) interdependencies between natural numbers and to understand the distribution of certain number groups in a visual way.

Therefore I want to ask mathematicians who read my study, to continue my work and to do more extensive graphical analyses of similar kind by using more advanced analysis-techniques and specialized computer software.

FIG. 7 :

Harry K. Hahn / 1.5.2006



K5 - Prime Number Spiral-Graph

Prime Number Factors	SD	K5	K5'	K5''
	4	13		
7x7	13	49	36	
	8	107	58	22
11x17	16	187	80	22
17x17	19	289	102	22
7x59	8	413	124	22
13x43	19	559	146	22
	16	727	168	22
7x31	17	917	190	22
	13	1129	212	22
29x47	13	1363	234	22
	17	1619	256	22
7x271	25	1897	278	22
13x13x13	19	2197	300	22
11x229	17	2519	322	22
7x409	19	2863	344	22
	16	3229	366	22
	17	3617	388	22
	13	4027	410	22
7x7x7x13	22	4459	432	22
17x17x17	17	4913	454	22
17x317	25	5389	476	22
7x29x29	28	5887	498	22
43x149	17	6407	520	22
	28	6949	542	22
11x383	16	7513	564	22
7x13x89	26	8099	586	22
	22	8707	608	22
	22	9337	630	22
7x1427	35	9989	652	22
	16	10663	674	22
37x307	19	11359	696	22
13x929	17	12077	718	22
7x1831	19	12817	740	22
		13579	762	22
		14363	784	22
		15169	806	22
		15997	828	22
		16847	850	22
		17719	872	22
		18613	894	22
		19529	916	22
		20467	938	22
		21427	960	22
		22409	982	22
		23413	1004	22
		24439	1026	22
		25487	1048	22
		26557	1070	22

This picture describes, how the „Prime Number Spiral Graphs“ shown in FIG. 6-A to 6-C run along the points of intersection of certain "Number Group Spiral Graphs" with the square root spiral (Einstein-Spiral)
For example : In the case of the Prime Number Spiral Graph K5 these are the points of intersection which are created by the intersection of the "Number Group Spiral Graphs" of the numbers divisible by 7, 11 and 17, with the square root spiral.

On this spiral shaped graphs (e.g. "Prime Number Spiral Graph K5" which are created along of these periodical appearing intersection points, new Prime Numbers are appearing in considerably quantities, which then form the starting points for new "Number Group Spiral Graphs" and which eventually lead to new "Prime Number Spiral Graphs".

Table 1 : Random analysis of the numbers of the Prime Number Spiral-Graphs (-sequences) **B3** , **Q3** and **P20-G1** (see FIG. 6-A to 6-C) in regards to their share in Prime Numbers. → **Spot checks carried out in the number area : 0 - 2 500 000 000**
Note : The selected Prime Number Spiral-Graphs contain a particular high share of Prime Numbers.

B3				
SD	Prime Factors	B3	B3'	B3"
8		17		
8		53	36	
8		107	54	18
17		179	72	18
17		269	90	18
17	13x29	377	108	18
8		503	126	18
17		647	144	18
17		809	162	18
26	23x43	989	180	18
17		1187	198	18
8	23x61	1403	216	18
17		1637	234	18
26		1889	252	18
17	17x127	2159	270	18
17		2447	288	18
17		2753	306	18
17	17x181	3077	324	18
17	13x263	3419	342	18
26		3779	360	18
17		4157	378	18
17	29x157	4553	396	18
26		4967	414	18
26		5399	432	18
26		5849	450	18
17		6317	468	18
17		6803	486	18
17		7307	504	18
26		7829	522	18
26		8369	540	18
26	79x113	8927	558	18
17	13x17x43	9503	576	18
17	23x439	10097	594	18
17		10709	612	18
17	17x23x29	11339	630	18
26		11987	648	18
17		12653	666	18
17		13337	684	18
17	53x47251	2504303	9486	18
26	17x29x5099	2513807	9504	18
26		2523329	9522	18
35		2532869	9540	18
26	107x23761	2542427	9558	18
17	53x179x269	2552003	9576	18
35	113x22669	2561597	9594	18
26		2571209	9612	18
17	17x797x1847	25025003	30006	18
26	23x23x47363	25055027	30024	18
35		25085069	30042	18
26	13x1931933	25115129	30060	18
26	2551x9857	25145207	30078	18
26		25175303	30096	18
26		25205417	30114	18
35		25235549	30132	18
26	191x1308919	250003529	94860	18
35	13x17x29x39023	250098407	94878	18
26	233x1073791	250193303	94896	18
35		250288217	94914	18
35		250383149	94932	18
44		250478099	94950	18
35	10253x24439	250573067	94968	18
35	23x10898611	250668053	94986	18
17		2500250003	300006	18
26	29x2731x31573	2500550027	300024	18
35	3793x659333	2500850069	300042	18
26	181x13818509	2501150129	300060	18
26		2501450207	300078	18
26	13x269x673x1063	2501750303	300096	18
26	43x58187219	2502050417	300114	18
35		2502350549	300132	18

Q3				
SD	Prime Factors	Q3	Q3'	Q3"
4		13		
10		37	24	
11		83	46	22
7		151	68	22
7		241	90	22
11		353	112	22
19		487	134	22
13		643	156	22
11		821	178	22
4		1021	200	22
10	11x113	1243	222	22
20		1487	244	22
16		1753	266	22
7	13x157	2041	288	22
11		2351	310	22
19		2683	332	22
13		3037	354	22
11		3413	376	22
13	37x103	3811	398	22
10		4231	420	22
20		4673	442	22
16	11x467	5137	464	22
16		5623	486	22
11		6131	508	22
19		6661	530	22
13		7213	552	22
29	13x599	7787	574	22
22	83x101	8383	596	22
10		9001	618	22
20	31x311	9641	640	22
7		10303	662	22
25		10987	684	22
20	11x1063	11693	706	22
10		12421	728	22
13		13171	750	22
20	73x191	13943	772	22
22		14737	794	22
19	103x151	15553	816	22
25	947x3221	3050287	11574	22
29	11x278353	3061883	11596	22
19	739x4159	3073501	11618	22
22	1319x2339	3085141	11640	22
29	233x13291	3096803	11662	22
31		3108487	11684	22
19	101x30893	3120193	11706	22
20	13x103x2339	3131921	11728	22
37	37x103x8017	30552787	36654	22
38		30589463	36676	22
25	503x60887	30626161	36698	22
34		30662881	36720	22
38	157x195539	30699623	36742	22
37	11x2794217	30736387	36764	22
31	31x197x5039	30773173	36786	22
38	743x41467	30809981	36808	22
34	113x2703137	305454481	115920	22
29	3407x89689	305570423	115942	22
46	103x1693x1753	305686387	115964	22
31	37x1303x6343	305802373	115986	22
38		305918381	116008	22
22	17419x17569	306034411	116030	22
28		306150463	116052	22
38		306266537	116074	22
49	13x235040599	3055527787	366654	22
47	137x499x44701	3055894463	366676	22
31	37x223x370411	3056261161	366698	22
46		3056627881	366720	22
47	2633x1161031	3056994623	366742	22
43	1433x2133539	3057361387	366764	22
43		3057728173	366786	22
47	3803x804127	3058094981	366808	22

P20 - G1				
SD	Prime Factors	G1	G1'	G1"
4		103		
11		173	70	
11		263	90	20
13		373	110	20
8		503	130	20
14		653	150	20
13		823	170	20
5		1013	190	20
8		1223	210	20
13		1453	230	20
11	13x131	1703	250	20
20		1973	270	20
13	31x73	2263	290	20
17	31x83	2573	310	20
14		2903	330	20
13		3253	350	20
14		3623	370	20
8		4013	390	20
13		4423	410	20
20	23x211	4853	430	20
11		5303	450	20
22	23x251	5773	470	20
17		6263	490	20
23	13x521	6773	510	20
13	67x109	7303	530	20
23		7853	550	20
17		8423	570	20
13		9013	590	20
20		9623	610	20
11		10253	630	20
13		10903	650	20
17	71x163	11573	670	20
14		12263	690	20
22		12973	710	20
14	71x193	13703	730	20
17	97x149	14453	750	20
13	13x1171	15223	770	20
11	67x239	16013	790	20
35	709x3947	2798423	10570	20
23	23x122131	2809013	10590	20
31	71x151x263	2819623	10610	20
23	53x53401	2830253	10630	20
26	13x218531	2840903	10650	20
31	71x40163	2851573	10670	20
29	617x4639	2862263	10690	20
38		2872973	10710	20
38		27855623	33370	20
38	71x392803	27889013	33390	20
31		27922423	33410	20
44	3803x7351	27955853	33430	20
41		27989303	33450	20
31	157x178489	28022773	33470	20
32	2161x12983	28056263	33490	20
44	83x338431	28089773	33510	20
53	2711x102523	277939853	105430	20
32		278045303	105450	20
40		278150773	105470	20
41		278256263	105490	20
44	97x643x4463	278361773	105510	20
40		278467303	105530	20
47	13x21428681	278572853	105550	20
47		278678423	105570	20
50		2778555623	333370	20
53	271x10254203	2778889013	333390	20
40		2779222423	333410	20
56	509x5460817	2779555853	333430	20
56		2779889303	333450	20
40	23x2539x47609	2780222773	333470	20
44	659x4219357	2780556263	333490	20
59	23x2741x44111	2780889773	333510	20

Note : Remarkable is the repeated occurrence of certain Prime-Factors (marked in red) !!
SD = sum of the digits

Table 2 : Analysis of the "Sums of the digits" which result from the numbers of the "Prime Number Sequences" shown in tables **6-A1** to **6-C1** (--> see columns **-SD-**)

2. Differential	**"Prime Number Sequences"** (see tables **6-A1** to **6-C1**) --> Sequences derived from Prime Number Spiral Graphs shown in FIG. 6-**A** to 6-**C**	**"Sum of the digits"-Sequence belonging to these Prime Number Sequences** (see columns **-SD-** in tables 6-A1 to 6-C1)	**Periodicity of "Sum of the digits" Sequence**
18	A2, A4, A6, A8, A10, A12..... C2, C4, C6, C8, C10, C12,.... 2 2 2	4,7,10,13,16,19,.... 3 3 3	3
	A1, A3, A5, A7, A9, A11..... C1, C3, C5, C7, C9, C11,....	2,5,8,11,14,17,....	3
	B1, B7, B13, B19...... 6 6 6	2,11,20,29,38,.... 9 9 9	9
	B2, B8, B14, B20,....	4,13,22,31,40,....	9
	B3, B9, B15, B21,....	8,17,26,35,44,....	9
	B4, B10, B16, B22,....	10,19,28,37,46,......	9
	B5, B11, B17, B23,....	5,14,23,32,41,....	9
	B6, B12, B18, B24,....	7,16,25,34,43,....	9

		D	**E**	**F**	**G**	**H**	**I**		
20	N -	D1		F3	G1		I1	1, 4, 7, 9,10,13,16,18,19,22,25,27,.... 3 3 2 1 3 3 2 1 3 3 2	3 , 3 , 2 , 1
	P -		E2			H2			
	N -	D3	E1		G3	H1	I3	1,3,4,7,10,12,13,16,19,21,22,....	3 , 3 , 2 , 1
	P -			F2					
	N -		E3	F1		H3		1,4,6,7,10,13,15,16,19,22,24,....	3 , 3 , 2 , 1
	P -	D2			G2		I2		
	N -		E2			H2		2,5,8,10,11,14,17,19,20,23,....	3 , 3 , 2 , 1
	P -	D3		F1	G3		I3		
	N -			F2				2,4,5,8,11,13,14,17,20,22,.....	3 , 3 , 2 , 1
	P -	D1	E3		G1	H3	I1		
	N -	D2			G2		I2	2,5,7,8,11,14,16,17,20,23,....	3 , 3 , 2 , 1
	P -		E1	F3		H1			

	J	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**		
22	J2	K1			N1	O3	P3	Q3	R3	S1	T3	2, 4, 7,10,11,13,16,19,20,22,25,.... 2 3 3 1 2 3 3 1 2 3 3	1 , 2 , 3 , 3
		K3	L1	M1	N3					S3		4,5,7,10,13,14,16,19,22,....	1 , 2 , 3 , 3
			L3	M3		O1	P1	Q1	R1		T1	4,7,8,10,13,16,17,19,22,....	1 , 2 , 3 , 3
			L2	M2								5,8,11,12,14,17,20,21,23,....	1 , 2 , 3 , 3
						O2	P2	Q2	R2		T2	2,5,6,8,11,14,15,17,20,....	1 , 2 , 3 , 3
		K2			N2					S2		5,8,9,11,14,17,18,20,23,....	1 , 2 , 3 , 3
	J3											6,7,9,12,15,16,18,21,24,...	1 , 2 , 3 , 3
	J1											4,6,9,12,13,15,18,21,22,....	1 , 2 , 3 , 3

Note : "Sum of the digits"-Sequences were created by ordering the "sums of the digits" occurring in a "prime number sequence" according to their value. --> See values in the columns **-SD-** in tables 6-D to 6-F

Note : „Sum of the digits“- Sequences were created by ordering the „sums of the digits“ occuring in the „prime number sequences“ according to their value. → see values in the columns **-SD-** in tables 6-A1 to 6-C1

Table 3 : Analysis of the periodic behaviour of the number endings in the "Prime Number Sequences" shown in tables **6-A1** to **6-C1**

"Number Endings"-Sequences contain the same numbers, but in different order !

2. Differential	Prime Number Sequences (see tables **6-A1** to **6-C1**) --> Sequences derived from "Prime Number Spiral Graphs" shown in FIG. 6-A to 6-C			Number Endings occurring in Sequence	"Number Endings"-Sequence belonging to these Prime Number Sequences (see tables 6-A1 to 6-C1)	Divisibility of the numbers in the "Prime-Number-Sequences" (Tab.6) by prime factor 3 and 5: Numbers divisible by 3	Numbers divisible by 5
18	**A2**,**A12**,A22,... A5,A15,A25,...	**B6**,B16,B26,... B9,B19,B29,...	**C4**,C14,C24,.. C7,C17,C27,..	...1 ; ...5 ; ...7 ;	7, 7, 5, 1, 5, 7, 7, 5, 1, 5,....	None	periodic every **5.** number (double) --> as indicated by arrows
	A4,A14,A24,... A7,A17,A27,...	**B8**,B18,B28,... B1,B11,B21,...	**C6**,C16,C26,.. C9,C19,C29,..	...1 ; ...3 ; ...7 ;	3, 3, 1, 7, 1, 3, 3, 1, 7, 1, ...	None	None
	A6,A16,A26,.. A9,A19,A29,..	**B10**,B20,B30,.. B3,B13,B23,...	**C8**,C18,C28,.. C1,C11,C31,..	...3 ; ...7 ; ...9 ;	7, 9, 9, 7, 3, 7, 9, 9, 7, 3, ...	None	None
	A8,A18,A28,... A1,A11,A21,...	**B2**,**B12**,B22,.. B5,B15,B25,...	**C10**,C20,C30,.. C3,C13,C23,..	...3 ; ...5 ; ...9 ;	3, 5, 5, 3, 9, 3, 5, 5, 3, 9, ...	None	periodic every **5.** number (double) --> as indicated by arrows
	A10,A20,A30,... A3,A13,A23,...	**B4**,B14,B24,... B7,B17,B27,...	**C2**,**C12**,C22,.. C5,C15,C25,..	...1 ; ...5 ; ...9 ;	9, 1, 1, 9, 5, 9, 1, 1, 9, 5, ...	None	periodic every **5.** number --> as indicated by red arrows

2. Differential		Prime Number Sequences	Number Endings occurring in Sequence	"Number Endings"-Sequence	Numbers divisible by 3	Numbers divisible by 5
20	N -	D3	...1	-----	periodic every **3.** number	None
	P -	D1			None	
	N -	G3	...3	-----	periodic every **3.** number	None
	P -	D2 G1			periodic every **3.** number (**only D2**)	
	N -	D2	...5	-----	None	all numbers
	P -	G2				
	N -	D1 G2	...7	-----	periodic every **3.** number (**only D1**)	None
	P -	D3			None	
	N -	G1	...9	-----	periodic every **3.** number	None
	P -	G3			None	
	N -	E1, E3, E5,....	...1 ; ...3 ; ...5 ; ...7 ; ...9	1, 5, 9, 3, 7, 1, 5, 9, 3, 7,.....	periodic every **3.** number	periodic every **5.** Number --> as indicated by red arrows
	N -	E2, E4, E6,....			None	
	P -	E1, E3 , E5,....			None	
	P -	E2 , E4 , E6 ,....			periodic every **3.** number	
	N -	H1, H3, H5,....			periodic every **3.** number	
	N -	H2, H4, H6,....			None	
	P -	H1, H3 , H5,....			None	
	P -	H2 , H4 , H6 ,....			periodic every **3.** number	
	N -	F1, F3, F5,....		7, 3, 9, 5, 1, 7, 3, 9, 5, 1,..... (Note opposite directions of sequences !)	periodic every **3.** number	periodic every **5.** Number --> as indicated by red arrows
	N -	F2, F4, F6,....			None	
	P -	F1, F3 , F5,....			None	
	P -	F2 , F4 , F6 ,....			periodic every **3.** number	
	N -	I1, I3, I5,....			periodic every **3.** number	
	N -	I2, I4, I6,....			None	
	P -	I1, I3 , I5,....			None	
	P -	I2 , I4 , I6 ,....			periodic every **3.** number	

2. Differential	J K L M N O P Q R S T	Number Endings occurring in Sequence	"Number Endings"-Sequence	Numbers divisible by 3	Numbers divisible by 5
22	P2 S2 T2	...1 ; ...5 ; ...9 ;	1, 9, 9, 1, 5, 1, 9, 9, 1, 5, ...	periodic every **3.** number	periodic every **5.** number --> as indicated by red arrows
	K1 N1			None	
	J1			periodic every **3.** number (double !)	
	J2 L3 M3 O3 Q1 R3	...3 ; ...7 ; ...9 ;	9, 7, 7, 9, 3, 9, 7, 7, 9, 3, ...	None	None
	K2 N2			periodic every **3.** number	
	K3 L1 M1 N3 O1 R1	...3 ; ...5 ; ...9 ;	5, 3, 3, 5, 9, 5, 3, 3, 5, 9, ...	None	periodic every **5.** number (double !) --> as indicated by arrows
	J3			periodic every **3.** number (double !)	
	L2 M2 O2 R2	...1 ; ...3 ; ...7 ;	3, 1, 1, 3, 7, 3, 1, 1, 3, 7, ...	periodic every **3.** number	None
	P1 Q3 S1 T1			None	
	P3 S3 T3	...1 ; ...5 ; ...7 ;	7, 5, 5, 7, 1, 7, 5, 5, 7, 1, ...	None	periodic every **5.** number (double !) --> as indicated by arrows
	Q2			periodic every **3.** number	

e.g. A15, A25 - "Prime Number Sequences" marked in green not shown in tables 6-A1 to 6-C1 (only for reference !)

e.g. N - D3 marking in red indicates that the "Prime Number Spiral Graphs" have a **negative** (N) rotation direction

P - D1 marking in blue indicates that the "Prime Number Spiral Graphs" have a **positive** (P) rotation direction

▽▼ indicates periodic occurence of numbers divisible by **5** in "Prime Number Sequences"

Table 4 : Periodic occurring Prime Factors in the „Prime-Number-Spiral-Graphs" (-number sequences) **B3** and **K5**
(see also FIG. 6-A / 6-C & 7 and Tables 6-A1 / 6-C1)

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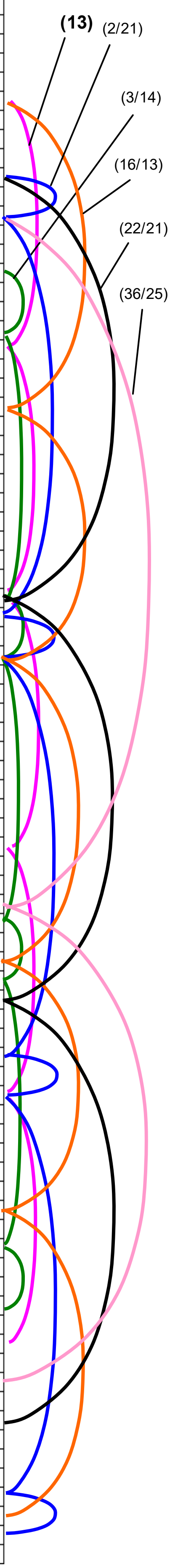
<-------- periodic occurrence of Individual Prime Factors → expressed through the number of spacings = (X)



61	43	29	23	17	13	Prime Factors of the none Prime-Numbers	sum of the digits	B3	B3'	B3''
								17		
							8	53	36	
							8	107	54	18
							17	179	72	18
							17	269	90	18
		29			13	13x29	17	377	108	18
							8	503	126	18
							17	647	144	18
							17	809	162	18
	43		23			23x43	26	989	180	18
							17	1187	198	18
61			23			23x61	8	1403	216	18
							17	1637	234	18
							26	1889	252	18
				17		17x127	17	2159	270	18
							17	2447	288	18
							17	2753	306	18
				17		17x181	17	3077	324	18
					13	13x263	17	3419	342	18
							26	3779	360	18
							17	4157	378	18
		29				29x157	17	4553	396	18
							26	4967	414	18
							26	5399	432	18
							26	5849	450	18
							17	6317	468	18
							17	6803	486	18
							17	7307	504	18
							26	7829	522	18
							26	8369	540	18
						79x113	26	8927	558	18
	43			17	13	13x17x43	17	9503	576	18
			23			23x439	17	10097	594	18
							17	10709	612	18
		29	23	17		17x23x29	17	11339	630	18
							26	11987	648	18
							17	12653	666	18
							17	13337	684	18
						101x139	17	14039	702	18
							26	14759	720	18
							26	15497	738	18
							17	16253	756	18
							17	17027	774	18
						103x173	26	17819	792	18
					13	13x1433	26	18629	810	18
							26	19457	828	18
						79x257	8	20303	846	18
61						61x347	17	21167	864	18
				17		17x1297	17	22049	882	18
						53x433	26	22949	900	18
		29				29x823	26	23867	918	18
				17		17x1459	17	24803	936	18
	43					43x599	26	25757	954	18
							26	26729	972	18
						53x523	26	27719	990	18
			23			23x1249	26	28727	1008	18
							26	29753	1026	18
			23		13	13x23x103	26	30797	1044	18
							26	31859	1062	18
							26	32939	1080	18
						101x337	17	34037	1098	18
							17	35153	1116	18
						131x277	26	36287	1134	18
		29				29x1291	26	37439	1152	18
							26	38609	1170	18
				17		17x2341	35	39797	1188	18
						131x313	8	41003	1206	18
							17	42227	1224	18
				17		17x2557	26	43469	1242	18
							26	44729	1260	18
					13	13x3539	17	46007	1278	18
							17	47303	1296	18
61						61x797	26	48617	1314	18
						199x251	35	49949	1332	18
	43					43x1193	26	51299	1350	18
							26	52667	1368	18
						191x283	17	54053	1386	18
							26	55457	1404	18
			23			23x2473	35	56879	1422	18
		29				29x2011	26	58319	1440	18
			23			23x23x113	35	59777	1458	18
							17	61253	1476	18

37	29	17	13	11	7	Prime Factors of the none Prime-Numbers	sum of the digits	K5	K5'	K5''
								13		
					7	7x7	13	49	36	
							8	107	58	22
		17		11		11x17	16	187	80	22
		17				17x17	19	289	102	22
					7	7x59	8	413	124	22
			13			13x43	19	559	146	22
							16	727	168	22
					7	7x31	17	917	190	22
							13	1129	212	22
	29					29x47	13	1363	234	22
							17	1619	256	22
					7	7x271	25	1897	278	22
			13			13x13x13	19	2197	300	22
				11		11x229	17	2519	322	22
					7	7x409	19	2863	344	22
							16	3229	366	22
							17	3617	388	22
							13	4027	410	22
			13		7	7x7x7x13	22	4459	432	22
		17				17x17x17	17	4913	454	22
		17				17x317	25	5389	476	22
	29				7	7x29x29	28	5887	498	22
						43x149	17	6407	520	22
							28	6949	542	22
				11		11x383	16	7513	564	22
			13		7	7x13x89	26	8099	586	22
							22	8707	608	22
							22	9337	630	22
					7	7x1427	35	9989	652	22
							16	10663	674	22
37						37x307	19	11359	696	22
			13			13x929	17	12077	718	22
					7	7x1831	19	12817	740	22
37						37x367	25	13579	762	22
						53x271	17	14363	784	22
				11	7	7x11x197	22	15169	806	22
		17				17x941	31	15997	828	22
		17				17x991	26	16847	850	22
	29		13			13x29x47	25	17719	872	22
					7	7x2659	19	18613	894	22
						59x331	26	19529	916	22
						97x211	19	20467	938	22
					7	7x3061	16	21427	960	22
							17	22409	982	22
			13			13x1801	13	23413	1004	22
							22	24439	1026	22
				11	7	7x11x331	26	25487	1048	22
							25	26557	1070	22
						43x643	28	27649	1092	22
					7	7x7x587	26	28763	1114	22
	29					29x1031	37	29899	1136	22
			13			13x2389	16	31057	1158	22
							17	32237	1180	22
		17			7	7x17x281	22	33439	1202	22
		17				17x2039	22	34663	1224	22
						149x241	26	35909	1246	22
					7	7x47x113	25	37177	1268	22
			13	11		11x13x269	28	38467	1290	22
							35	39779	1312	22
							10	41113	1334	22
					7	7x6067	25	42469	1356	22
						163x269	26	43847	1378	22
							22	45247	1400	22
					7	7x59x113	31	46669	1422	22
			13			13x3701	17	48113	1444	22
						43x1153	34	49579	1466	22
						223x229	19	51067	1488	22
37	29				7	7x7x29x37	26	52577	1510	22
				11		11x4919	19	54109	1532	22
							25	55663	1554	22
37		17	13		7	7x13x17x37	26	57239	1576	22
		17				17x3461	31	58837	1598	22
							22	60457	1620	22
							26	62099	1642	22
					7	7x9109	25	63763	1664	22
							28	65449	1686	22
							26	67157	1708	22
			13		7	7x13x757	37	68887	1730	22
							25	70639	1752	22
	29			11		11x29x227	17	72413	1774	22
							22	74209	1796	22

Note : the number of "spacings" (or lines) which lie between two successive prime factors of the same value, corresponds to the number of successive spiral windings of the Square-Root-Spiral ("Einstein-Spiral"), which lie between the two numbers which contain these prime factors (see FIG. 6-A / 6-C) !

Table 5-A : Periodic occurring Prime Factors in the „Prime-Number-Spiral-Graphs" (-number sequences) **Q3** and **P20-G1**
(see also FIG. 6-B / 6-C and Tables 6-B1 / 6-C1)

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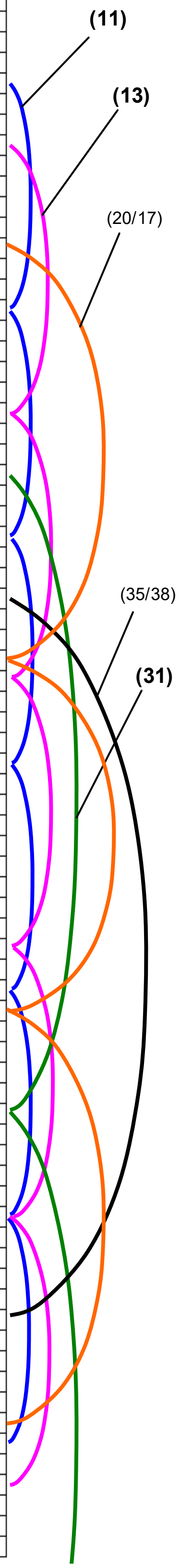


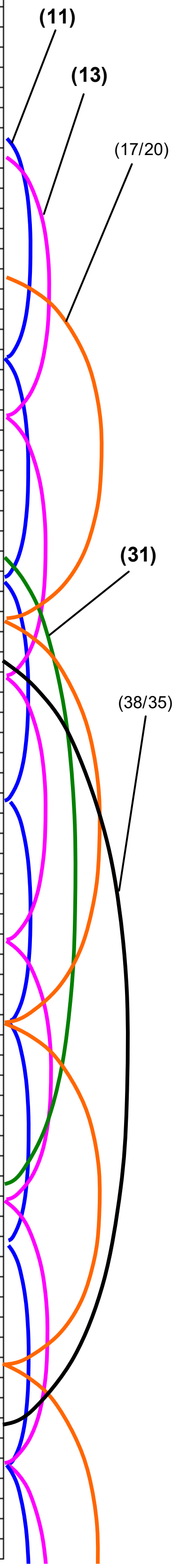
some Prime Factors presented in tabular form: 73	37	31	13	11	<-------- periodic occurrence of Individual Prime Factors → expressed through the number of spacings = (X)	Prime Factors of the none Prime-Numbers	sum of the digits	Q3	Q3'	Q3''
								13		
							10	37	24	
							11	83	46	22
							7	151	68	22
							7	241	90	22
							11	353	112	22
							19	487	134	22
							13	643	156	22
							11	821	178	22
							4	1021	200	22
				11		11x113	10	1243	222	22
							20	1487	244	22
							16	1753	266	22
			13			13x157	7	2041	288	22
							11	2351	310	22
							19	2683	332	22
							13	3037	354	22
							11	3413	376	22
	37					37x103	13	3811	398	22
							10	4231	420	22
							20	4673	442	22
				11		11x467	16	5137	464	22
							16	5623	486	22
							11	6131	508	22
							19	6661	530	22
							13	7213	552	22
			13			13x599	29	7787	574	22
						83x101	22	8383	596	22
							10	9001	618	22
		31				31x311	20	9641	640	22
							7	10303	662	22
							25	10987	684	22
				11		11x1063	20	11693	706	22
							10	12421	728	22
							13	13171	750	22
73						73x191	20	13943	772	22
							22	14737	794	22
						103x151	19	15553	816	22
	37					37x443	20	16391	838	22
			13			13x1327	16	17251	860	22
							16	18133	882	22
							20	19037	904	22
							28	19963	926	22
				11		11x1901	13	20911	948	22
							20	21881	970	22
						89x257	22	22873	992	22
							28	23887	1014	22
							20	24923	1036	22
							25	25981	1058	22
							16	27061	1080	22
							20	28163	1102	22
							28	29287	1124	22
			13			13x2341	13	30433	1146	22
							11	31601	1168	22
				11		11x11x271	22	32791	1190	22
	37					37x919	10	34003	1212	22
						167x211	20	35237	1234	22
							25	36493	1256	22
						107x353	25	37771	1278	22
						89x439	20	39071	1300	22
		31				31x1303	19	40393	1322	22
							22	41737	1344	22
							11	43103	1366	22
							22	44491	1388	22
						197x233	19	45901	1410	22
			13	11		11x13x331	20	47333	1432	22
							34	48787	1454	22
							16	50263	1476	22
						191x271	20	51761	1498	22
							19	53281	1520	22
73						73x751	22	54823	1542	22
						113x499	29	56387	1564	22
							31	57973	1586	22
							28	59581	1608	22
							11	61211	1630	22
	37					37x1699	25	62863	1652	22
				11		11x5867	25	64537	1674	22
						107x619	20	66233	1696	22
			13			13x5227	28	67951	1718	22
							31	69691	1740	22
							20	71453	1762	22
							22	73237	1784	22

Note : the number of "spacings" (or lines) which lie between two successive prime factors of the same value, corresponds to the number of successive spiral windings of the Square-Root-Spiral (Einstein-Spiral) , which lie between the two numbers which contain these prime factors (see FIG. 6-B / 6-C) !

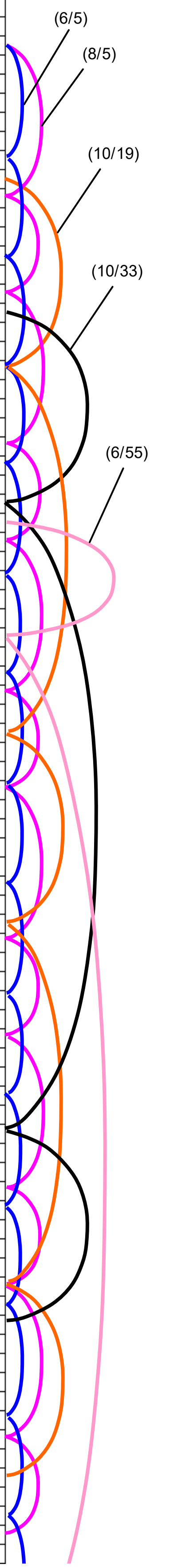
some Prime Factors presented in tabular form: 73	67	31	23	13	<-------- periodic occurrence of Individual Prime Factors → expressed through the number of spacings = (X)	Prime Factors of the none Prime-Numbers	sum of the digits	G1	G1'	G1''
								103		
							11	173	70	
							11	263	90	20
							13	373	110	20
							8	503	130	20
							14	653	150	20
							13	823	170	20
							5	1013	190	20
							8	1223	210	20
							13	1453	230	20
				13		13x131	11	1703	250	20
							20	1973	270	20
73		31				31x73	13	2263	290	20
		31				31x83	17	2573	310	20
							14	2903	330	20
							13	3253	350	20
							14	3623	370	20
							8	4013	390	20
							13	4423	410	20
			23			23x211	20	4853	430	20
							11	5303	450	20
			23			23x251	22	5773	470	20
							17	6263	490	20
				13		13x521	23	6773	510	20
	67					67x109	13	7303	530	20
							23	7853	550	20
							17	8423	570	20
							13	9013	590	20
							20	9623	610	20
							11	10253	630	20
							13	10903	650	20
						71x163	17	11573	670	20
							14	12263	690	20
							22	12973	710	20
						71x193	14	13703	730	20
						97x149	17	14453	750	20
				13		13x1171	13	15223	770	20
	67					67x239	11	16013	790	20
							20	16823	810	20
						127x139	22	17653	830	20
							17	18503	850	20
							23	19373	870	20
			23			23x881	13	20263	890	20
		31				31x683	14	21173	910	20
		31	23			23x31x31	8	22103	930	20
							13	23053	950	20
							11	24023	970	20
							11	25013	990	20
						53x491	13	26023	1010	20
				13		13x2081	17	27053	1030	20
						157x179	14	28103	1050	20
							22	29173	1070	20
						53x571	14	30263	1090	20
						137x229	17	31373	1110	20
							13	32503	1130	20
73						73x461	20	33653	1150	20
						97x359	20	34823	1170	20
							13	36013	1190	20
							17	37223	1210	20
							23	38453	1230	20
							22	39703	1250	20
							23	40973	1270	20
				13		13x3251	17	42263	1290	20
							22	43573	1310	20
						83x541	20	44903	1330	20
			23			23x2011	20	46253	1350	20
							22	47623	1370	20
			23			23x2131	17	49013	1390	20
							14	50423	1410	20
							22	51853	1430	20
						151x353	14	53303	1450	20
							26	54773	1470	20
							22	56263	1490	20
							29	57773	1510	20
		31				31x1913	20	59303	1530	20
		31		13		13x31x151	22	60853	1550	20
							17	62423	1570	20
							14	64013	1590	20
						137x479	16	65623	1610	20
						109x617	23	67253	1630	20
							26	68903	1650	20
							22	70573	1670	20
						127x569	20	72263	1690	20
							29	73973	1710	20
							22	75703	1730	20
73						73x1061	26	77453	1750	20
						227x349	23	79223	1770	20
							13	81013	1790	20
			23	13		13x23x277	23	82823	1810	20
							26	84653	1830	20
			23			23x3761	22	86503	1850	20
	67					67x1319	29	88373	1870	20

Table 5-B : Periodic occurring Prime Factors in the „Prime-Number-Spiral-Graphs" (-number sequences) **S1** and **B33**
(see also FIG. 6-A / 6-C and Tables 6-A1 / 6-C1)

some Prime Factors presented in tabular form: 73	37	31	13	11	<-------- periodic occurence of individual Prime Factors --> expressed through the numbers of spacings = (X)	Prime Factors of the None-Prime-Numbers	sum of the digits	S1	S1'	S1"
							2	11		
							4	31	20	
							10	73	42	22
							11	137	64	22
							7	223	86	22
							7	331	108	22
							11	461	130	22
							10	613	152	22
							22	787	174	22
							20	983	196	22
							4	1201	218	22
				11		11x131	10	1441	240	22
			13			13x131	11	1703	262	22
							25	1987	284	22
							16	2293	306	22
							11	2621	328	22
							19	2971	350	22
							13	3343	372	22
	37					37x101	20	3737	394	22
							13	4153	416	22
							19	4591	438	22
							11	5051	460	22
				11		11x503	16	5533	482	22
							16	6037	504	22
							20	6563	526	22
			13			13x547	10	7111	548	22
							22	7681	570	22
							20	8273	592	22
							31	8887	614	22
						89x107	19	9523	636	22
							11	10181	658	22
							16	10861	680	22
		31				31x373	16	11563	702	22
				11		11x1117	20	12287	724	22
							10	13033	746	22
	37					37x373	13	13801	768	22
							20	14591	790	22
73						73x211	13	15403	812	22
			13			13x1249	19	16237	834	22
							20	17093	856	22
							25	17971	878	22
						113x167	25	18871	900	22
							29	19793	922	22
						89x233	19	20737	944	22
				11		11x1973	13	21703	966	22
							20	22691	988	22
						137x173	13	23701	1010	22
							19	24733	1032	22
						107x241	29	25787	1054	22
							25	26863	1076	22
							25	27961	1098	22
			13			13x2237	20	29081	1120	22
							10	30223	1142	22
							22	31387	1164	22
							20	32573	1186	22
	37			11		11x37x83	22	33781	1208	22
						157x223	10	35011	1230	22
							20	36263	1252	22
							25	37537	1274	22
							25	38833	1296	22
							11	40151	1318	22
							19	41491	1340	22
							22	42853	1362	22
		31				31x1427	20	44237	1384	22
			13			13x3511	22	45643	1406	22
						103x457	19	47071	1428	22
				11		11x11x401	20	48521	1450	22
							34	49993	1472	22
							25	51487	1494	22
							11	53003	1516	22
							19	54541	1538	22
							13	56101	1560	22
	37					37x1559	29	57683	1582	22
						101x587	31	59287	1604	22
							19	60913	1626	22
73						73x857	20	62561	1648	22
							16	64231	1670	22
			13	11		11x13x461	25	65923	1692	22
						239x283	29	67637	1714	22
						173x401	28	69373	1736	22
						83x857	13	71131	1758	22
							20	72911	1780	22



some Prime Factors presented in tabular form: 61	43	29	13	11	<-------- periodic occurence of individual Prime Factors --> expressed through the numbers of spacings = (X)	Prime Factors of the None-Prime-Numbers	sum of the digits	B33	B33'	B33"
							17	89		
							8	107	18	
			13	11		11x13	8	143	36	18
							17	197	54	18
							17	269	72	18
							17	359	90	18
							17	467	108	18
							17	593	126	18
				11		11x67	17	737	144	18
		29				29x31	26	899	162	18
			13			13x83	17	1079	180	18
							17	1277	198	18
							17	1493	216	18
				11		11x157	17	1727	234	18
							26	1979	252	18
			13			13x173	17	2249	270	18
	43					43x59	17	2537	288	18
							17	2843	306	18
							17	3167	324	18
		29		11		11x11x29	17	3509	342	18
						53x73	26	3869	360	18
						31x137	17	4247	378	18
							17	4643	396	18
			13			13x389	17	5057	414	18
				11		11x499	26	5489	432	18
							26	5939	450	18
	43					43x149	17	6407	468	18
61						61x113	26	6893	486	18
			13			13x569	26	7397	504	18
							26	7919	522	18
				11		11x769	26	8459	540	18
						71x127	17	9017	558	18
						53x181	26	9593	576	18
61						61x167	17	10187	594	18
							26	10799	612	18
				11		11x1039	17	11429	630	18
			13			13x929	17	12077	648	18
							17	12743	666	18
		29				29x463	17	13427	684	18
						71x199	17	14129	702	18
						31x479	26	14849	720	18
			13	11		11x13x109	26	15587	738	18
						59x277	17	16343	756	18
							17	17117	774	18
							26	17909	792	18
							26	18719	810	18
				11		11x1777	26	19547	828	18
							17	20393	846	18
		29				29x733	17	21257	864	18
			13			13x13x131	17	22139	882	18
							17	23039	900	18
							26	23957	918	18
				11		11x31x73	26	24893	936	18
							26	25847	954	18
			13			13x2063	26	26819	972	18
							26	27809	990	18
							26	28817	1008	18
				11		11x2713	26	29843	1026	18
						67x461	26	30887	1044	18
	43					43x743	26	31949	1062	18
							17	33029	1080	18
							17	34127	1098	18
			13			13x2711	17	35243	1116	18
				11		11x3307	26	36377	1134	18
							26	37529	1152	18
							35	38699	1170	18
							35	39887	1188	18
		29	13			13x29x109	17	41093	1206	18
				11		11x3847	17	42317	1224	18
	43					43x1013	26	43559	1242	18
							26	44819	1260	18
						31x1487	26	46097	1278	18
						83x571	26	47393	1296	18
						53x919	26	48707	1314	18
				11		11x4549	17	50039	1332	18
			13			13x59x67	26	51389	1350	18
							26	52757	1368	18
		29				29x1867	17	54143	1386	18
							26	55547	1404	18
				11		11x5179	35	56969	1422	18
			13			13x4493	26	58409	1440	18
						131x457	35	59867	1458	18



Note : the number of "spacings" (or lines) which lie between two successive prime factors of the same value, corresponds to the number of successive spiral windings of the Square-Root-Spiral ("Einstein-Spiral"), which lie between the two numbers which contain these prime factors (see FIG. 6-A / 6-C) !

9 The Number Spiral - by Robert Sachs → www.numberspiral.com

The following chapter shows extracts from the analysis of the " Number Spiral ", carried out by Mr. Robert Sachs.

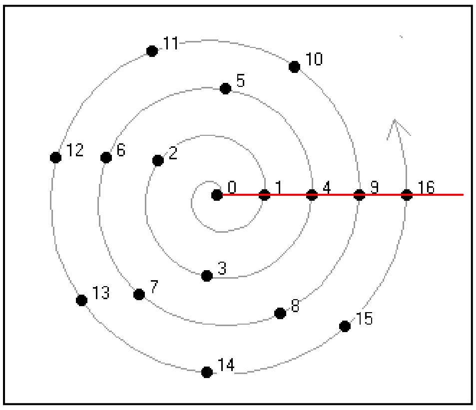


FIG. NS-1 : Number Spiral

9.1 Introduction

Number spirals are very simple. To make one, we just write the non-negative integers on a ribbon and roll it up with zero at the center. The trick is to arrange the spiral so all the perfect squares (1, 4, 9, 16, etc.) line up in a row on the right side: → **see Figure NS-1 & NS-2**

If we continue winding for a while and zoom out a bit, the result looks like shown on the right .

If we zoom out even further and remove everything except the dots that indicate the locations of integers, we get **Figure NS-3** below. It shows 2026 dots :

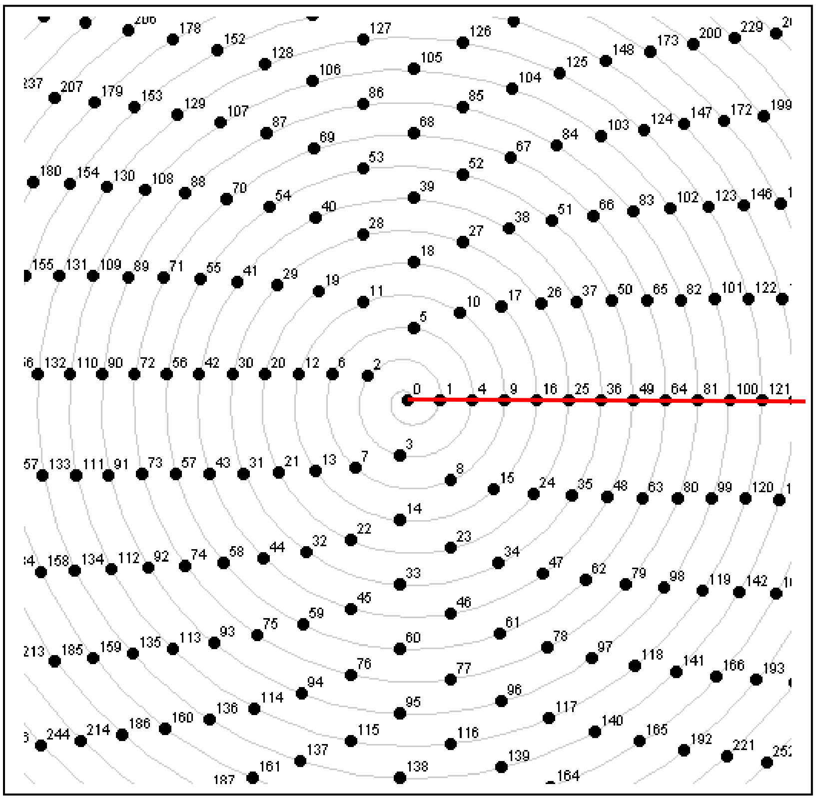


FIG. NS-2 : Number Spiral with perfect squares lined-up on a straight line

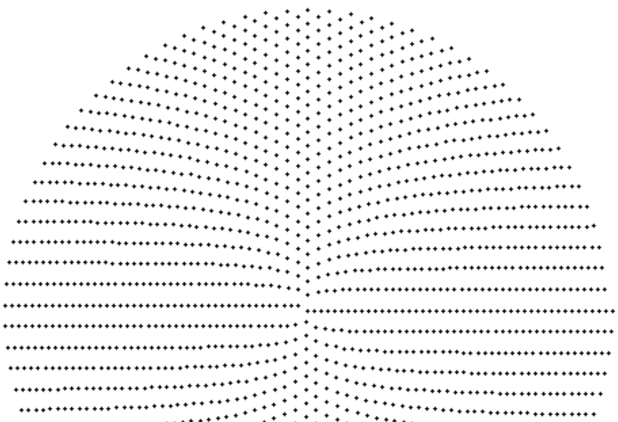


FIG. NS-3 : Number Spiral
→ dots indicating 2026 integers

Let's try making the primes darker than the non-primes:
→ **see Figure NS-4**

The primes clearly seem to cluster along certain curves. (see image below)

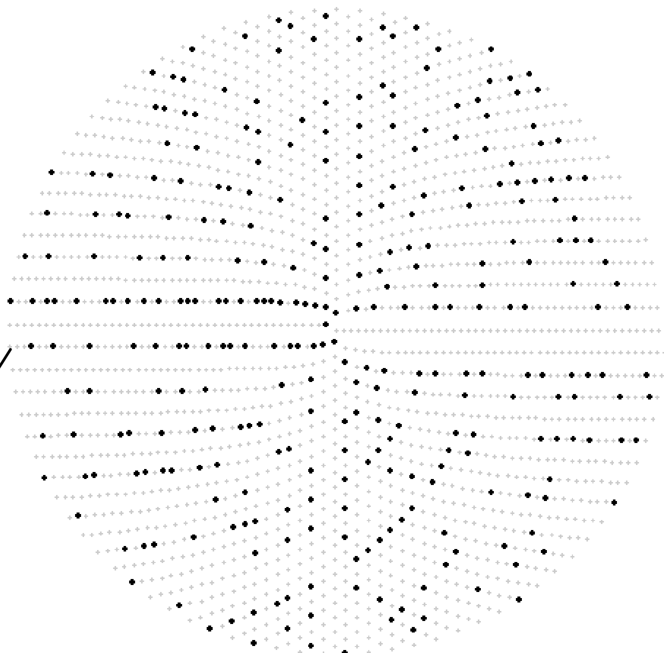


FIG. NS-4 : Prime Numbers cluster along defined curves

"Curve P-1" contains the numbers 5, 11, 19, 29, 41, 55, 71... which result in the quadratic polynomials :
$x^2 + 3x + 1$ or $x^2 + 5x + 5$ or $x^2 + 7x + 11$ etc.

"Curve P+1" contains the numbers 3, 7, 13, 21, 31, 43, 57... which result in the quadratic polynomials :
$x^2 + x + 1$ or $x^2 + 3x + 3$ or $x^2 + 5x + 7$ etc.

Let's zoom out even further to get a better look. The following number spiral shows all the primes that occur within the first 46,656 non-negative integers. (For clarity, non-primes have been left out.)

→ **see Figure NS-5**

It is evident that prime numbers concentrate on certain curves which run to the northwest and southwest, like the curve marked by the blue arrow.

In the following we'll investigate these patterns and try to make sense out of them.

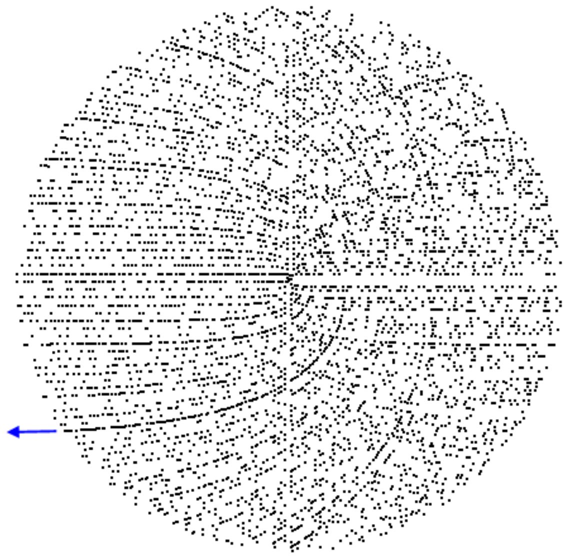


FIG. NS-5 : Shows the Positions of the first 46656 Prime Number

9.2 Product Curves

On the previous images we saw that primes tend to line up in curves on the number spiral. In fact, the whole spiral is made of curves of this kind, and every integer belongs to an infinite number of them.

The simplest curve of this type (the one with least curvature) is the line of perfect squares, marked here in blue → **see Figure NS-6**

For convenience, I'll call this line "curve S" for "squares."

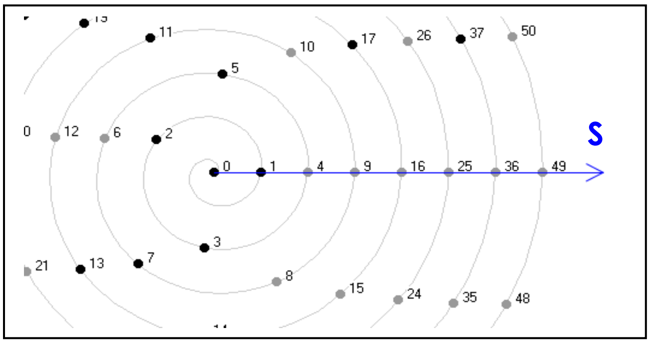


FIG. NS-6: "Curve S" – Line of perfect squares on the Number Spiral

Here's another example → **see Figure NS-7**

The factors of numbers on this second curve are 1 x 2, 2 x 3, 3 x 4, etc. The difference between factors is always 1. Such numbers are called " pronics ", so I'll name this second line "Curve P".

In curve S, the difference between factors is zero. In curve P, it is one. And there are other such curves at distances of 1, 2, 4, 6 , 9, 12,..." units" to the original curves P or S .

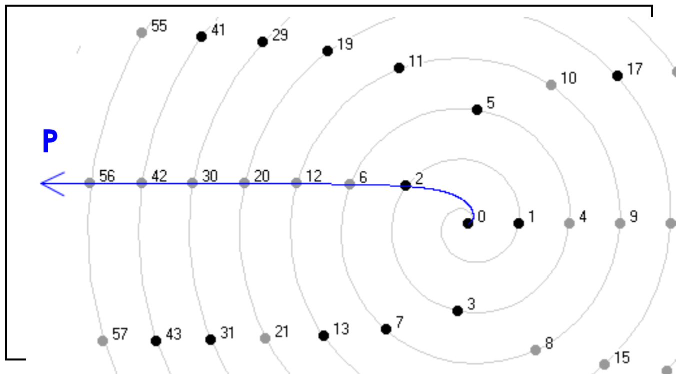


FIG. NS-7: "Curve P" – Line of " pronics " with difference between factors of numbers on this curve is always 1

For example curve P-6 , which is at a distance of "6 units" to P , contains the numbers 50, 66, 84, 104, 126,…. In this sequence the factors of the numbers are 5 x 10 , 6 x 11 , 7 x 12 , 8 x 13 , ...etc. , which corresponds to a difference of 5 between the factors of a number and to the difference of 1 between the factors of two successive numbers.

Or curve S-1 , which is at a distance of one unit to S , contains the numbers 15, 24, 35, 48, 63, 80,…. In this number sequence the factors of the numbers are 3 x 5 , 4 x 6 , 5 x 7 , 6 x 8 , 7 x 9 , 8 x 10 etc. , which corresponds to a difference of 2 between the factors of a number and to the difference of 1 between the factors of two successive numbers.

We can continue this way forever, increasing the distance to the curves P and S and finding new curves.

On the lefthand side in **Figure NS-8** the first ten such curves are shown:

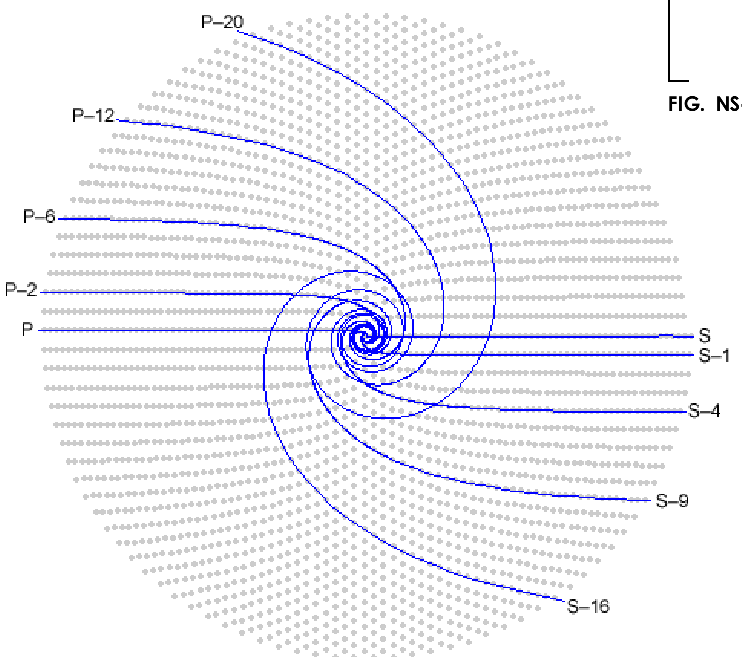


FIG. NS-8 : Shows the first ten " pronics – curves "

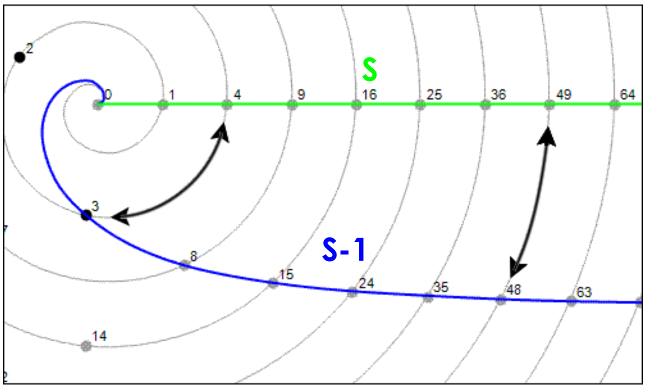


FIG. NS-9 : Distances between the numbers on curve S and curve S-1 are **constant** measured along the spiral

9.3 Offset Curves

At the bottom of the previous page we saw a picture of the first ten product curves (pronics – curves). Product curves are important because every possible way of multiplying one number by another is represented on one of them.

But it turns out that they are only a special case of a more general phenomenon. Their properties are due to the fact that they are located at fixed distances from defined angles. As we will see, other curves which are located at fixed distances from other defined angles have similar properties.
To illustrate these ideas, let's look at product curve S – 1, shown above in blue (see **Figure NS-9**). It is located at a fixed distance from angle zero, shown in green.

At first glance the green and blue lines appear to converge at the left. But if we measure the distance between them using the spiral itself as our tape measure, the lines are always **one unit** apart. The black arrows show how to do this. The left arrow stretches between 3 and 4, a distance of one. The right arrow stretches between 48 and 49, also a distance of one. Even when the blue curve is at zero, it's separated (for measuring purposes) from the green curve by the piece of the spiral that runs from zero to one. No matter where we measure, the distance is always **1**.
I call a constant distance of this sort an offset. When a curve is offset from an angle line, I call it an offset curve.
If we zoom out far enough, offset curves look straight. Some of them have so little curvature that we barely have to zoom. For instance:

The blue lines in **Figure NS-10** show the first offset curves (with offset 0) of the angle lines which have a rotational angle of 0, 1/64, 1/32, 1/16, 1/8, and 1/4 in reference to curve S (curve which contains the perfect squares). Note : one full rotation = 1.

Offset curves are important because some of them are composite. When I say that a curve is composite I mean that all the integers on them (except for the first, which is always zero, and the second, which may be prime) are non-prime. Moreover, we can predict the factorization of any integer on such curves just from knowing its location.
Every rational angle has composite offset curves.
When I say "rational angle," I mean an angle line whose measurement in rotations (in reference to curve S) is a rational number: 1/2 rotation, 1/3 rotation, 1473/2076 rotation, etc.

Figure NS-11, for example shows the 1/3 angle line marked in green (at an angle of 120 degrees in reference to curve S) :

And **Figure NS-12** on the right shows a few of its offset curves:

The blue offset curves are composite. (To avoid misunderstanding, let me say again that the first integer on a composite curve is zero, the second may or may not be prime, and the rest are composite.)

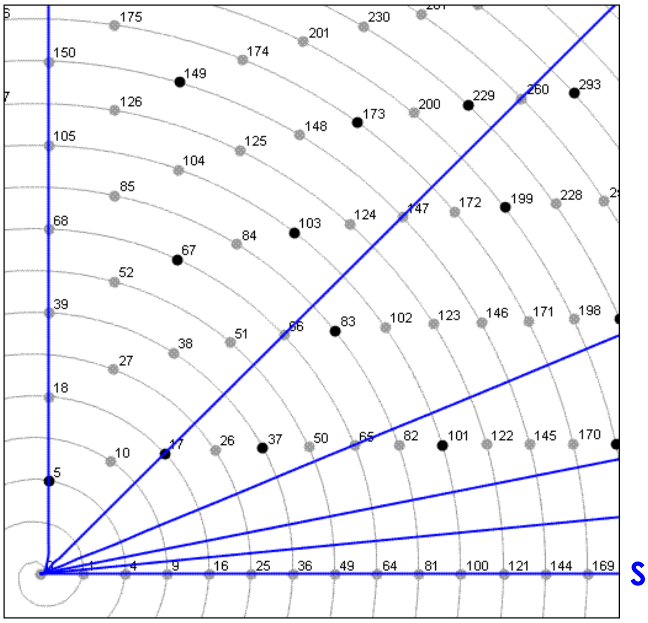


FIG. NS-10 : First 5 Offset-Curves at rotational angles of 1/64, 1/32, 1/16, 1/8 and 1/4 in reference to Curve S

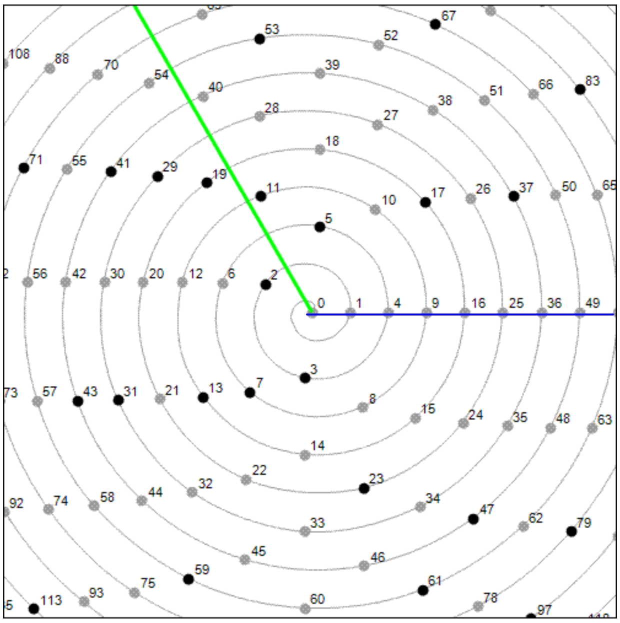


FIG. NS-11 : Angle Line with rotational angle 1/3 (120 degrees)

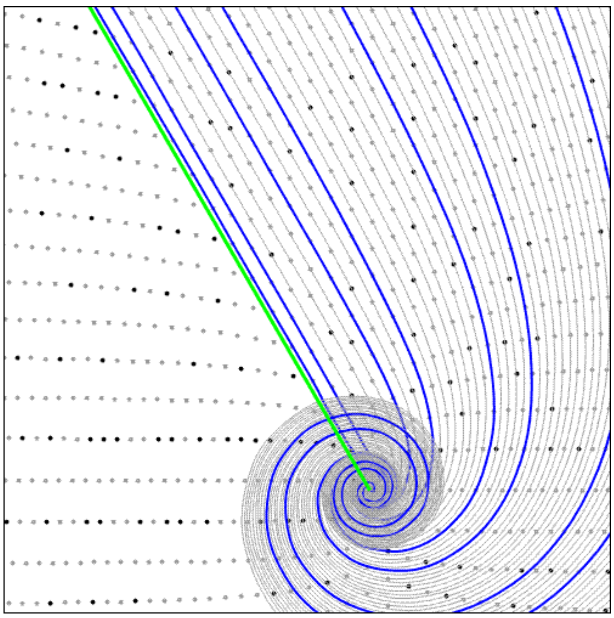


FIG. NS-12 : Offset-Curves of angle line with angle 1/3 (120 deg.)

As I said a moment ago, every rational angle has blue offset curves of this type. Here in **Figure NS-13** for example the angle 29/33 is shown :

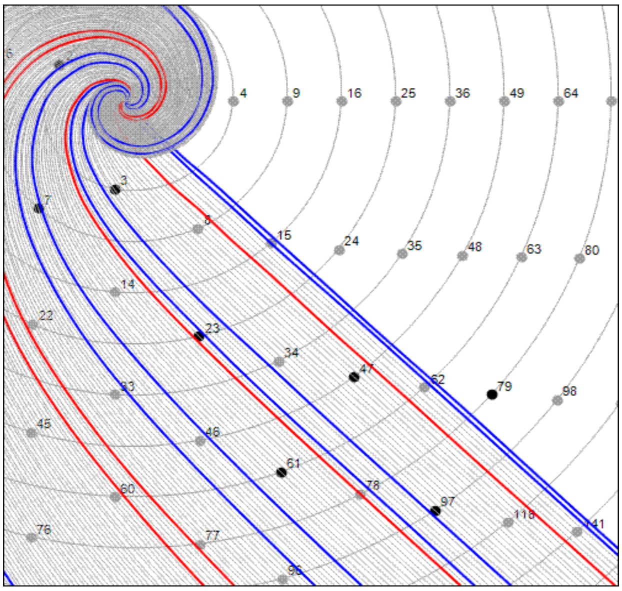


FIG. NS-13 : Shows the Offset-Curves of angle 29/33

In this Figure you see something new : red curves. Like blue curves, red ones are composite. But red and blue sequences are composite for different reasons, so I distinguish them by color.

One last example: **Figure NS-14** shows the first few offset curves for angle 1/4 (90 deg.) → see below on the right :

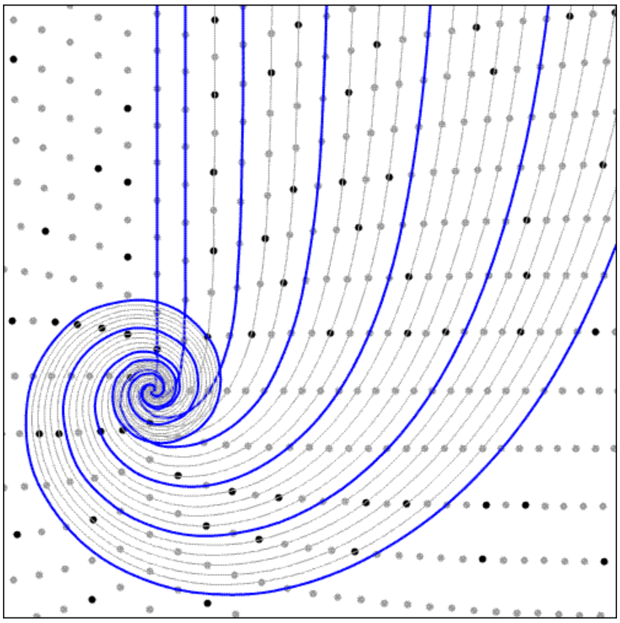


FIG. NS-14 : Shows the Offset-Curves of angle 1/4 (90 deg.)

I have a special reason for including angle 1/4 in my analysis. When I first wrote my website in 2003, I didn't realize that every rational angle has composite curves. Therefore I was puzzled by the prominent columns of non-prime numbers that march north and south from the center of the spiral, and I published **Figure NS-15** as a mystery to be solved later (see below) :

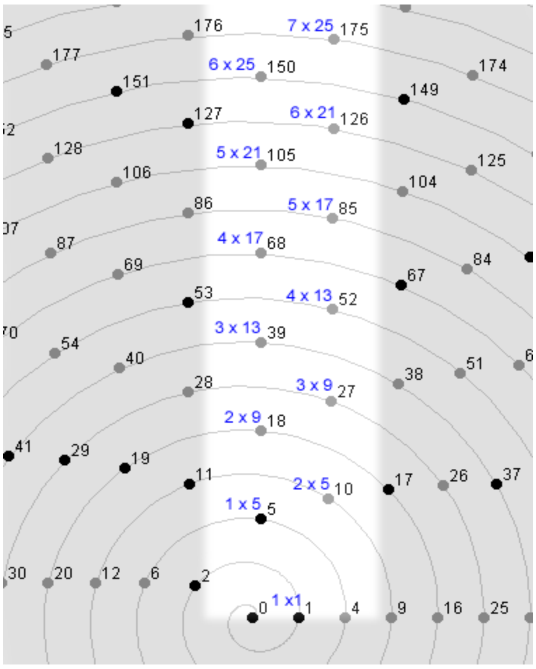


FIG. NS-15 : A column of non-primes runs north & south

9.4 Quadratic Polynomials (Quadratic Functions)

Every offset curve can be generated by a quadratic polynomial of the form:

$$y = ax^2 + bx + c$$

Where *y* is the number that appears on the graph; *a*, *b*, and *c* are constants that define the curve; and *x* is the index of the integer on the curve. In order for a quadratic function to be an offset curve, *a* must be a perfect square. In order for a function to be a *composite* offset curve, *c* must be zero. In other words, the function for a composite offset curve looks like this:

$$y = ax^2 + bx$$

Of course *a* must be a perfect square in the above equation, because composite curves are a subset of offset curves. For composite offset curves of angle *n*/*d* where the denominator is even:

$$a = (d/2)^2$$
$$b = n$$
$$\text{offset} = (n/d)^2$$

For angles with odd denominators, multiply the numerator and denominator by two and use the above formulas. The correspondence between angles and coefficients creates an **orderly pattern** on the graph. This can be seen in the illustration on the right **Figure NS-16** , which shows all the composite curves for denominator 6 with the numerator ranging from –9 to 9 inclusive:

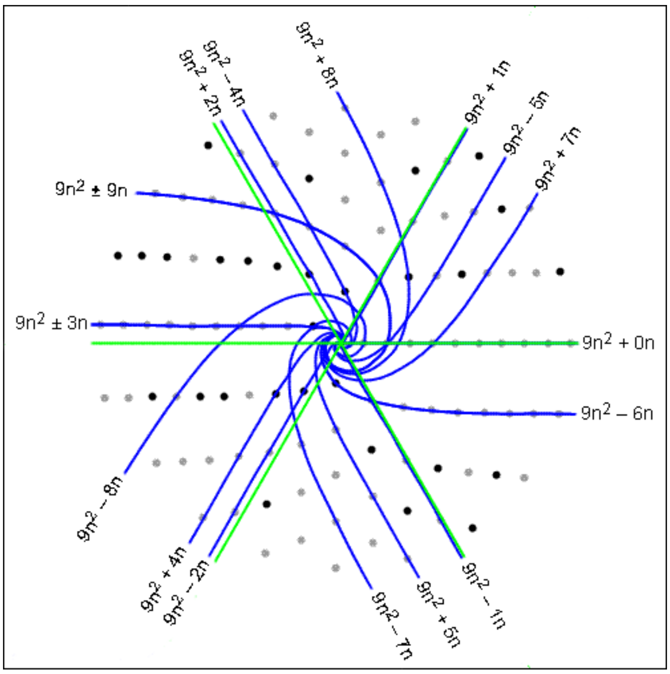


FIG. NS-16: offset curves are **orderly** distributed quadratic polynomials

9.5 Primes

Now that we have adopted a convention for naming the curves on the number spiral, let's look again at how primes are distributed.

As shown in **Figure NS-17** the densest concentrations of primes seem to occur mainly in curves whose offsets are prime.

When we look at a graph that shows only primes, like **Figure NS-17**, the left-facing curves are much more pronounced than the right-facing curves. However, when we look at a graph that shows all the integers, like **Figure NS-3** the left-facing and right-facing curves are equally salient. The main reason for this seems to be that on the left side, primes can occur only in curves with odd offsets. On the right side, primes can occur in curves with both odd and even offsets. It would be interesting to investigate empirically whether equal quantities of primes occur on both sides.

→ Interresting would also be an analysis of the circular appearances of prime numbers in this diagram !! (→ comment from Harry K. Hahn)

In FIG. NS-17 the first 46656 Prime Numbers are shown.

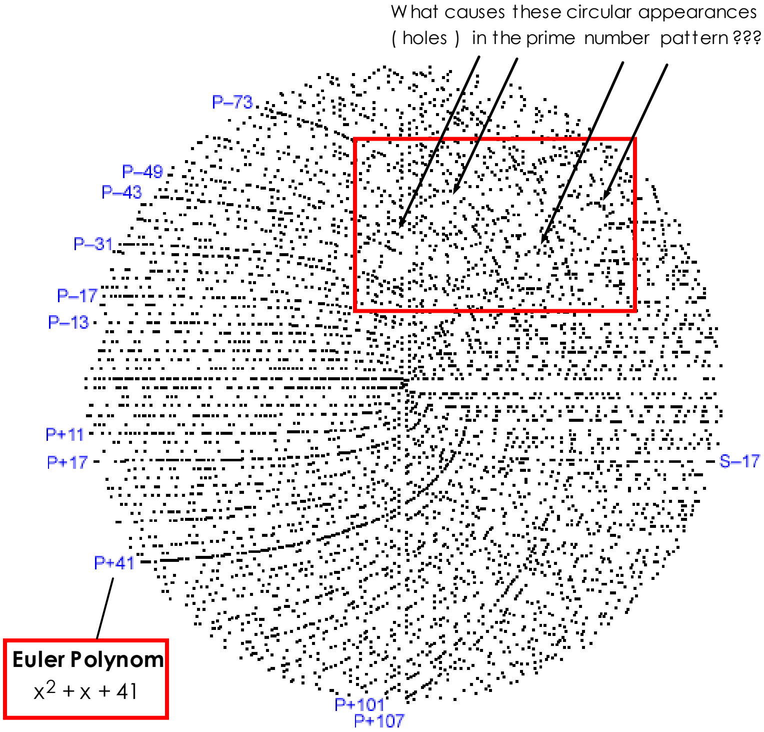


FIG. NS-17 : Prime Numbers seem to occur mainly in curves whose offsets are prime

Here a comparison of the Number Spiral with the Ulam-Spiral → **see Figure NS-18**

Each diagonal of the Ulam spiral corresponds to a particular curve on the number spiral.

However, the Ulam spiral makes two diagonals out of each curve by allowing both ends of a diagonal to grow in opposite directions and placing alternate members of the sequence at either end. Moreover, the two halves of each diagonal do not usually line up with each other. This sounds terribly confusing in words, but as shown in **Figure NS-18**, it's really pretty simple.

I've labeled only a few diagonals in this Figure to illustrate the pattern, the correspondence extends infinitely:

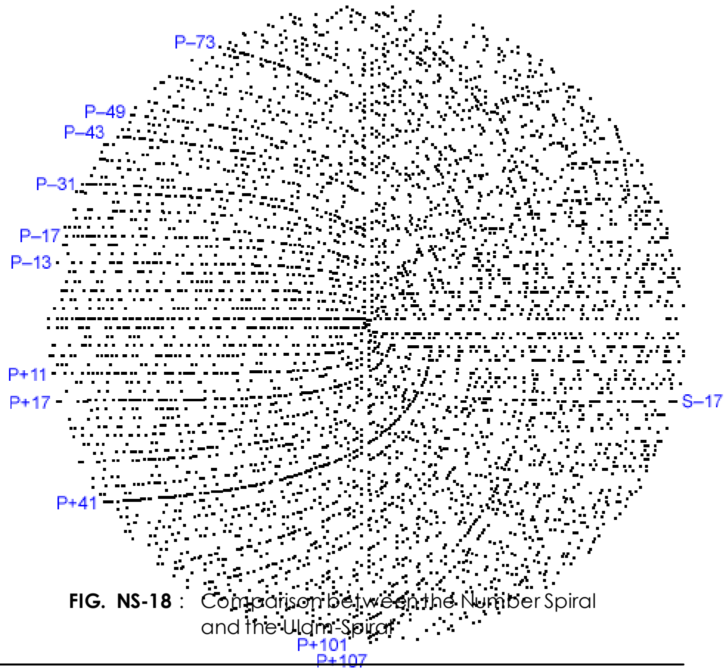


FIG. NS-18 : Comparison between the Number Spiral and the Ulam-Spiral

9.6 FORMULAS

To convert from polar coordinates to Windows screen coordinates (in which y increases from top to bottom, unlike conventional graphs):

$$x = r\cos(2\pi\theta)$$
$$y = -r\sin(2\pi\theta)$$

…: theta is in rotations; …d y must be scaled and …lated before using them …r screen display

To plot the spiral, including both the thin gray coiled line and the integers on it: :

$$r = \sqrt{n}$$
$$\theta = \sqrt{n}$$

Note: theta is in rotations

To plot offset curves:

$$r = \sqrt{an^2 + bn + c}$$
$$\theta = r - n\sqrt{a}$$

To derive coefficients *a, b, c* of a quadratic formula from three successive integers *i, j, k* in a quadratic sequence:

$$a = \frac{i - 2j + k}{2}$$
$$b = j - i - 3a$$
$$c = i - a - b$$

For more information about these formulas including their derivation, see Method of Common Differences.

To convert between a composite offset curve of angle *n/d* (measured in rotations) and its related quadratic function $y = ax^2 + bx$:

$$a = \left(\frac{d}{2}\right)^2$$
$$b = n$$
$$offset = \left(\frac{n}{d}\right)^2$$
$$d = 2\sqrt{a}$$

To factor an integer on a composite offset curve:

→ $y = x(ax + b)$

FIG. 8 :

Comparison between " Square Root Spiral " and " Number Spiral " (by R. Sachs)

PRONICS - Spiral Graphs :

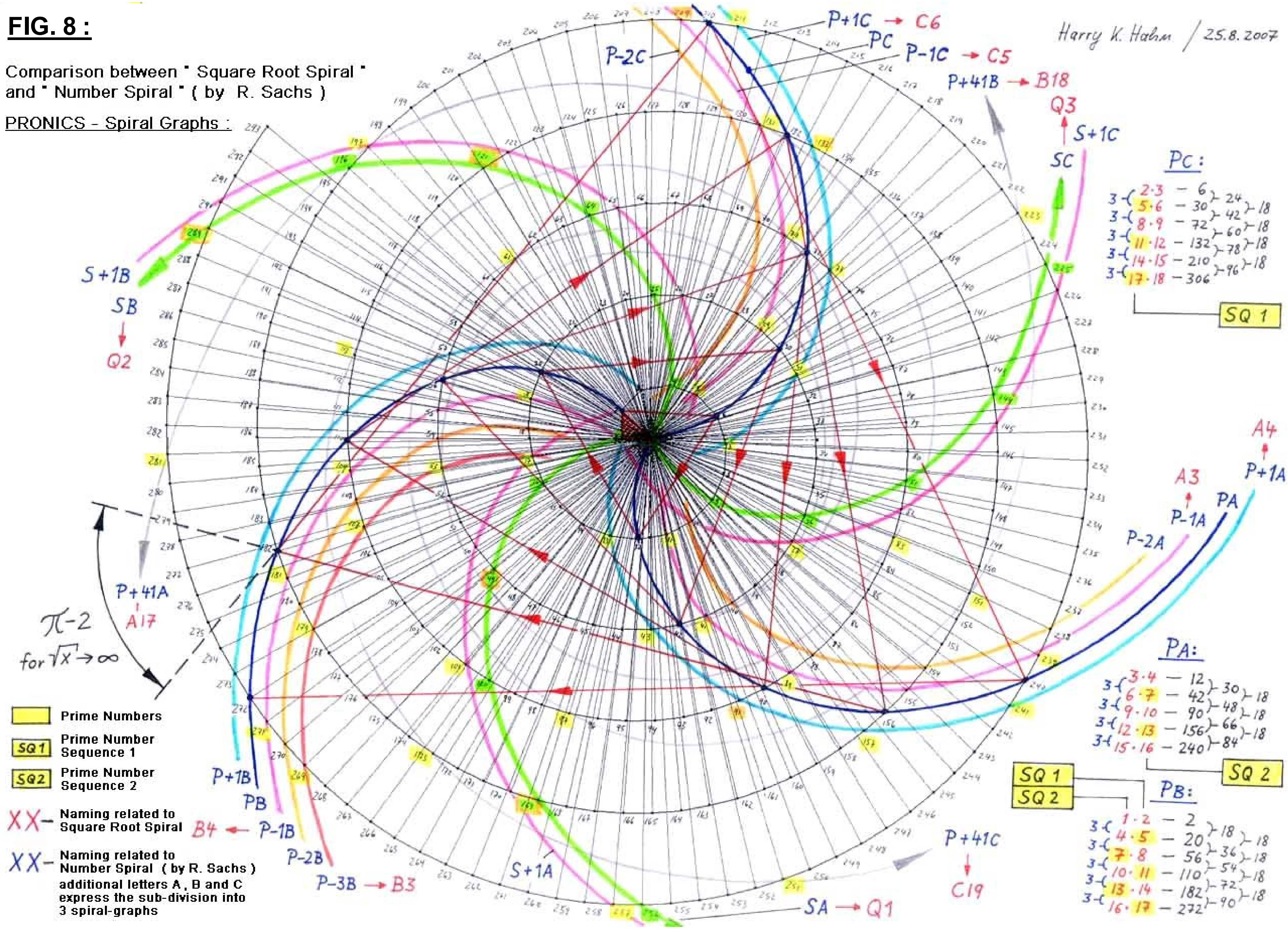


Table 7-A : Comparison of the arrangement of the "PRONICS-Spiral Graphs" (product curves) S , P , S+1 and P-1 on the "Number Spiral" & "Square Root Spiral" (see FIG.: NS-8 at page 29 and FIG.: 8 at page 33)

NUMBER SPIRAL → **SQUARE ROOT SPIRAL**

factors	SD	S	S'	S"
1x1	1	1		
2x2	4	4	3	
3x3	9	9	5	2
4x4	5	16	7	2
5x5	7	25	9	2
6x6	9	36	11	2
7x7	13	49	13	2
8x8	10	64	15	2
	9	81	17	2
	1	100	19	2
	4	121	21	2
	9	144	23	2
	16	169	25	2
	16	196	27	2
	9	225	29	2
	13	256	31	2
	19	289	33	2
	9	324	35	2
	10	361	37	2
	4	400	39	2
	9	441	41	2
	16	484	43	2
	16	529	45	2
	18	576	47	2
	13	625	49	2

factors	SD	SA (Q1)	SA'	SA"
1x1	1	1		
4x4	7	16	15	
7x7	13	49	33	18
10x10	1	100	51	18
13x13	16	169	69	18
16x16	13	256	87	18
19x19	10	361	105	18
22x22	16	484	123	18
	13	625	141	18
	19	784	159	18
	16	961	177	18
	13	1156	195	18
	19	1369	213	18
	7	1600	231	18
	22	1849	249	18
	10	2116	267	18
	7	2401	285	18
	13	2704	303	18
	10	3025	321	18
	16	3364	339	18
	13	3721	357	18
	19	4096	375	18
	25	4489	393	18
	13	4900	411	18
	19	5329	429	18

factors	SD	SB (Q2)	SB'	SB"
2x2	4	4		
5x5	7	25	21	
8x8	10	64	39	18
11x11	4	121	57	18
14x14	16	196	75	18
17x17	19	289	93	18
20x20	4	400	111	18
23x23	16	529	129	18
	19	676	147	18
	13	841	165	18
	7	1024	183	18
	10	1225	201	18
	13	1444	219	18
	16	1681	237	18
	19	1936	255	18
	13	2209	273	18
	7	2500	291	18
	19	2809	309	18
	13	3136	327	18
	16	3481	345	18
	19	3844	363	18
	13	4225	381	18
	16	4624	399	18
	10	5041	417	18
	22	5476	435	18

factors	SD	SC (Q3)	SC'	SC"
3x3	9	9		
6x6	9	36	27	
9x9	9	81	45	18
12x12	9	144	63	18
15x15	9	225	81	18
18x18	9	324	99	18
21x21	9	441	117	18
24x24	18	576	135	18
	18	729	153	18
	9	900	171	18
	18	1089	189	18
	18	1296	207	18
	9	1521	225	18
	18	1764	243	18
	9	2025	261	18
	9	2304	279	18
	9	2601	297	18
	18	2916	315	18
	18	3249	333	18
	9	3600	351	18
	27	3969	369	18
	18	4356	387	18
	18	4761	405	18
	18	5184	423	18
	18	5625	441	18

NUMBER SPIRAL → **SQUARE ROOT SPIRAL**

SD	S+1	S+1'	S+1"
5	5		
1	10	5	
8	17	7	2
8	26	9	2
10	37	11	2
5	50	13	2
11	65	15	2
10	82	17	2
2	101	19	2
5	122	21	2
10	145	23	2
8	170	25	2
17	197	27	2
10	226	29	2
14	257	31	2
11	290	33	2
10	325	35	2
11	362	37	2
5	401	39	2
10	442	41	2
17	485	43	2
8	530	45	2
19	577	47	2
14	626	49	2
20	677	51	2

SD	S+1A	S+1A'	S+1A"
2	2		
8	17	15	
5	50	33	18
2	101	51	18
8	170	69	18
14	257	87	18
11	362	105	18
17	485	123	18
14	626	141	18
20	785	159	18
17	962	177	18
14	1157	195	18
11	1370	213	18
8	1601	231	18
14	1850	249	18
11	2117	267	18
8	2402	285	18
14	2705	303	18
11	3026	321	18
17	3365	339	18
14	3722	357	18
20	4097	375	18
17	4490	393	18
14	4901	411	18
11	5330	429	18
26	5777	447	18

SD	S+1B	S+1B'	S+1B"
5	5		
8	26	21	
11	65	39	18
5	122	57	18
17	197	75	18
11	290	93	18
5	401	111	18
8	530	129	18
20	677	147	18
14	842	165	18
8	1025	183	18
11	1226	201	18
14	1445	219	18
17	1682	237	18
20	1937	255	18
5	2210	273	18
8	2501	291	18
11	2810	309	18
14	3137	327	18
17	3482	345	18
20	3845	363	18
14	4226	381	18
17	4625	399	18
11	5042	417	18
23	5477	435	18

SD	S+1C	S+1C'	S+1C"
1	1		
1	10	9	
10	37	27	18
10	82	45	18
10	145	63	18
10	226	81	18
10	325	99	18
10	442	117	18
19	577	135	18
10	730	153	18
10	901	171	18
10	1090	189	18
19	1297	207	18
10	1522	225	18
19	1765	243	18
10	2026	261	18
10	2305	279	18
10	2602	297	18
19	2917	315	18
10	3250	333	18
10	3601	351	18
19	3970	369	18
19	4357	387	18
19	4762	405	18
19	5185	423	18

factors	SD	P	P'	P"
1x2	2	2		
2x3	6	6	4	
3x4	3	12	6	2
4x5	2	20	8	2
5x6	3	30	10	2
6x7	6	42	12	2
7x8	11	56	14	2
8x9	9	72	16	2
9x10	9	90	18	2
10x11	2	110	20	2
11x12	6	132	22	2
	12	156	24	2
	11	182	26	2
	3	210	28	2
	6	240	30	2
	11	272	32	2
	9	306	34	2
	9	342	36	2
	11	380	38	2
	6	420	40	2
	12	462	42	2
	11	506	44	2
	12	552	46	2
	6	600	48	2
	11	650	50	2

factors	SD	PA	PA'	PA"
0x1	0	0		
3x4	3	12	12	
6x7	6	42	30	18
9x10	9	90	48	18
12x13	12	156	66	18
15x16	6	240	84	18
18x19	9	342	102	18
21x22	12	462	120	18
24x25	6	600	138	18
27x28	18	756	156	18
30x31	12	930	174	18
	6	1122	192	18
	9	1332	210	18
	12	1560	228	18
	15	1806	246	18
	9	2070	264	18
	12	2352	282	18
	15	2652	300	18
	18	2970	318	18
	12	3306	336	18
	15	3660	354	18
	9	4032	372	18
	12	4422	390	18
	15	4830	408	18
	18	5256	426	18

factors	SD	PB	PB'	PB"
1x2	2	2		
4x5	2	20	18	
7x8	11	56	36	18
10x11	2	110	54	18
13x14	11	182	72	18
16x17	11	272	90	18
19x20	11	380	108	18
22x23	11	506	126	18
25x26	11	650	144	18
28x29	11	812	162	18
31x32	20	992	180	18
	11	1190	198	18
	11	1406	216	18
	11	1640	234	18
	20	1892	252	18
	11	2162	270	18
	11	2450	288	18
	20	2756	306	18
	11	3080	324	18
	11	3422	342	18
	20	3782	360	18
	11	4160	378	18
	20	4556	396	18
	20	4970	414	18
	11	5402	432	18

factors	SD	PC	PC'	PC"
2x3	6	6		
5x6	3	30	24	
8x9	9	72	42	18
11x12	6	132	60	18
14x15	3	210	78	18
17x18	9	306	96	18
20x21	6	420	114	18
23x24	12	552	132	18
26x27	9	702	150	18
29x30	15	870	168	18
32x33	12	1056	186	18
	9	1260	204	18
	15	1482	222	18
	12	1722	240	18
	18	1980	258	18
	15	2256	276	18
	12	2550	294	18
	18	2862	312	18
	15	3192	330	18
	12	3540	348	18
	18	3906	366	18
	15	4290	384	18
	21	4692	402	18
	9	5112	420	18
	15	5550	438	18

SD	P-1	P-1'	P-1"
5	5		
2	11	6	
10	19	8	2
11	29	10	2
5	41	12	2
10	55	14	2
8	71	16	2
17	89	18	2
10	109	20	2
5	131	22	2
11	155	24	2
10	181	26	2
11	209	28	2
14	239	30	2
10	271	32	2
8	305	34	2
8	341	36	2
19	379	38	2
14	419	40	2
11	461	42	2
10	505	44	2
11	551	46	2
23	599	48	2
19	649	50	2
8	701	52	2

SD	P-1A (A3)	P-1A'	P-1A"
2	11		
5	41	30	
17	89	48	18
11	155	66	18
14	239	84	18
8	341	102	18
11	461	120	18
23	599	138	18
17	755	156	18
20	929	174	18
5	1121	192	18
8	1331	210	18
20	1559	228	18
14	1805	246	18
17	2069	264	18
11	2351	282	18
14	2651	300	18
26	2969	318	18
11	3305	336	18
23	3659	354	18
8	4031	372	18
11	4421	390	18
23	4829	408	18
17	5255	426	18
29	5699	444	18

SD	P-1B (B4)	P-1B'	P-1B"
10	19		
10	55	36	
10	109	54	18
10	181	72	18
10	271	90	18
19	379	108	18
10	505	126	18
19	649	144	18
10	811	162	18
19	991	180	18
19	1189	198	18
10	1405	216	18
19	1639	234	18
19	1891	252	18
10	2161	270	18
19	2449	288	18
19	2755	306	18
19	3079	324	18
10	3421	342	18
19	3781	360	18
19	4159	378	18
19	4555	396	18
28	4969	414	18
10	5401	432	18
19	5851	450	18

SD	P-1C (C5)	P-1C'	P-1C"
5	5		
11	29	24	
8	71	42	18
5	131	60	18
11	209	78	18
8	305	96	18
14	419	114	18
11	551	132	18
8	701	150	18
23	869	168	18
11	1055	186	18
17	1259	204	18
14	1481	222	18
11	1721	240	18
26	1979	258	18
14	2255	276	18
20	2549	294	18
17	2861	312	18
14	3191	330	18
20	3539	348	18
17	3905	366	18
23	4289	384	18
20	4691	402	18
8	5111	420	18
23	5549	438	18

XX Prime Number Sequence **SQ1** (blue)

XX Prime Number Sequence **SQ2** (red)

Prime Numbers (yellow)

Numbers not divisible by 2, 3 or 5 (orange)

Table 7-B : Prime Number Sequences derived from Spiral Graphs **P+41** (Euler Polynom → see FIG.: NS-17 at page 32) and **P+41A** , **P+41B** and **P+41C** (see FIG.: 8 at page 33)

NUMBER SPIRAL

Euler Polynom → P+41

SD	P+41	P+41'	P+41''
5	41		
7	43	2	
11	47	4	2
8	53	6	2
7	61	8	2
8	71	10	2
11	83	12	2
16	97	14	2
5	113	16	2
5	131	18	2
7	151	20	2
11	173	22	2
17	197	24	2
7	223	26	2
8	251	28	2
11	281	30	2
7	313	32	2
14	347	34	2
14	383	36	2
7	421	38	2
11	461	40	2
8	503	42	2
16	547	44	2
17	593	46	2
11	641	48	2
16	691	50	2
14	743	52	2
23	797	54	2
16	853	56	2
11	911	58	2
17	971	60	2
7	1033	62	2
17	1097	64	2
11	1163	66	2
7	1231	68	2
5	1301	70	2
14	1373	72	2
16	1447	74	2
11	1523	76	2
8	1601	78	2
16	1681	80	2
17	1763	82	2
20	1847	84	2

SQUARE ROOT SPIRAL

SD	prime factors	P+41A	P+41A'	P+41A''
5		41		
8		53	12	
11		83	30	18
5		131	48	18
17		197	66	18
11		281	84	18
14		383	102	18
8		503	120	18
11		641	138	18
23		797	156	18
17		971	174	18
11		1163	192	18
14		1373	210	18
8		1601	228	18
20		1847	246	18
5		2111	264	18
17		2393	282	18
20		2693	300	18
5		3011	318	18
17		3347	336	18
11		3701	354	18
14		4073	372	18
17	14 / 27	4463	390	18
20		4871	408	18
23	1 / 42	5297	426	18
17		5741	444	18
11		6203	462	18
23	41 x 163	6683	480	18
17	43 x 167	7181	498	18
29	43 x 179	7697	516	18
14		8231	534	18
26		8783	552	18
20	47 x 199	9353	570	18
23		9941	588	18
17	53 x 199	10547	606	18
11		11171	624	18
14		11813	642	18
17		12473	660	18
11		13151	678	18
23	61 x 227	13847	696	18
17		14561	714	18
20	41 x 373	15293	732	18

SD	prime factors	P+41B	P+41B'	P+41B''
7		43		
7		61	18	
16		97	36	18
7		151	54	18
7		223	72	18
7		313	90	18
7		421	108	18
16		547	126	18
16		691	144	18
16		853	162	18
7	14 / 27	1033	180	18
7		1231	198	18
16		1447	216	18
16	41 x 41	1681	234	18
16		1933	252	18
7		2203	270	18
16	47 x 53	2491	288	18
25		2797	306	18
7		3121	324	18
16		3463	342	18
16	14 / 33	3823	360	18
7		4201	378	18
25		4597	396	18
7		5011	414	18
16		5443	432	18
25	71 x 83	5893	450	18
16		6361	468	18
25	41 x 167	6847	486	18
16		7351	504	18
25		7873	522	18
16	47 x 179	8413	540	18
25		8971	558	18
25		9547	576	18
7		10141	594	18
16		10753	612	18
16		11383	630	18
7	53 x 227	12031	648	18
25		12697	666	18
16		13381	684	18
16		14083	702	18
16	113 x 131	14803	720	18
16		15541	738	18

SD	prime factors	P+41C	P+41C'	P+41C''
5		41		
11		47	6	
8		71	24	18
5		113	42	18
11		173	60	18
8		251	78	18
14		347	96	18
11		461	114	18
17		593	132	18
14		743	150	18
11		911	168	18
17	27 / 14	1097	186	18
5		1301	204	18
11		1523	222	18
17	41 x 43	1763	240	18
5	43 x 47	2021	258	18
20		2297	276	18
17		2591	294	18
14		2903	312	18
11	53 x 61	3233	330	18
17		3581	348	18
23		3947	366	18
11	61 x 71	4331	384	18
17		4733	402	18
14		5153	420	18
20	1 / 42	5591	438	18
17		6047	456	18
14		6521	474	18
11		7013	492	18
17		7523	510	18
14	83 x 97	8051	528	18
29		8597	546	18
17		9161	564	18
23		9743	582	18
11		10343	600	18
17	97 x 113	10961	618	18
23		11597	636	18
11		12251	654	18
17		12923	672	18
14		13613	690	18
11		14321	708	18
17	41 x 367	15047	726	18

Prime Numbers (yellow)

Numbers not divisible by 2, 3 or 5 (orange)

Table 7-C : Quadratic Polynomials of the "Prime-Number-Spiral-Graphs" **P+41** (Number Spiral) and **P+41A** to **P+41C** {(Square Root Spiral) --> 2. Differential = 18 }

	Spiral Graph	Number Sequence of Spiral Graph	Quadratic Polynomial 1 (calculated with the first 3 numbers of the given sequence)	Quadratic Polynomial 2 (calculated with 3 numbers starting with the 2. Number of the sequence)	Quadratic Polynomial 3 (calculated with 3 numbers starting with the 3. Number of the sequence)	Quadratic Polynomial 4 (calculated with 3 numbers starting with the 4. Number of the sequence)
Number Spiral	P+41	41 , 43 , 47 , 53 , 61 , 71 , ...	$f_1(x) = 1x^2 - 1x + 41$	$f_2(x) = 1x^2 + 1x + 41$	$f_3(x) = 1x^2 + 3x + 43$	$f_4(x) = 1x^2 + 5x + 47$
Square Root Spiral	P+41A	41 , 53 , 83 , 131 , 197 , 281 , ...	$f_1(x) = 9x^2 - 15x + 47$	$f_2(x) = 9x^2 + 3x + 41$	$f_3(x) = 9x^2 + 21x + 53$	$f_4(x) = 9x^2 + 39x + 83$
	P+41B	43 , 61 , 97 , 151 , 223 , 313 , ...	$f_1(x) = 9x^2 - 9x + 43$	$f_2(x) = 9x^2 + 9x + 43$	$f_3(x) = 9x^2 + 27x + 61$	$f_4(x) = 9x^2 + 45x + 97$
	P+41C	41 , 47 , 71 , 113 , 173 , 251 , ...	$f_1(x) = 9x^2 - 21x + 53$	$f_2(x) = 9x^2 - 3x + 41$	$f_3(x) = 9x^2 + 15x + 47$	$f_4(x) = 9x^2 + 33x + 71$

10 Comparison of the Ulam Spiral, Number Spiral and Square Root Spiral

Especially interesting should be a direct **comparison of** the **Ulam-Spiral** the **Number Spiral** and the **Square Root Spiral** in regards to the distribution of certain number-groups e.g. Square-Numbers , Pronics , Prime-Numbers etc.

For that purpose I have produced another diagram of the Square Root Spiral which shows some " reference graphs " which will make this comparison easier.

→ see FIG. 8 - "Comparison between Square Root Spiral & Number Spiral "

Besides **FIG. 8** (on page **33**), the following two images from Mr. Sach's analysis (→ chapter **9**) should be used for the above mentioned comparison :

Image **NS-8 : Number Spiral -** (Pronics Curves) - page **29**
Image **NS-18 : Ulam Spiral -** (Comparison with Number Spiral) - page **32**

→ These 3 images : **FIG 8 , NS-8** and **NS-18** of the Square Root Spiral , Number Spiral and Ulam Spiral , show the arrangement of the same number sequences or "Reference Graphs"

The main reference graphs are the ones which contain the square numbers. These are either named **S** on the Ulam- and Number-Spiral (FIG. NS-18 / NS-8), or **SA, SB, SC** on the Square Root Spiral (drawn in green color in FIG 8). (Note : the original naming of graphs SA, SB, SC on the Square Root Spiral is actually Q1, Q2, Q3 → see FIG.1)

The difference in the distribution of the square numbers is as follows :

Number Spiral : Square Numbers are located on **1** straight graph
Ulam Spiral : Square Numbers are located on **2** straight graphs
Square Root Spiral : Square Numbers are located on **3** spiral graphs

The difference is caused through the fact, that on the Number Spiral every wind only contains one square number , whereas the winds of the Ulam Spiral and Square Root Spiral contain either 2 or 3 successive Square Numbers of the Square Number Sequence per wind.

Or put in other words, the Number Spiral has the tightest spiral winding, followed by the Ulam Spiral, which has a wider winding and then followed by the Square Root Spiral which has the widest winding of all three spirals. There are of course further spiral variants possible with 4, 5, 6 and more successive square numbers per wind of the spiral, which then would have 4, 5, 6 or more (spiral) graphs on which the square numbers would be located on.

It would definitely be interesting to analyse all these spiral variants in regards to the distribution of certain number groups like pronics or prime numbers etc. !!

Beside the graphs SA, SB, SC which contain the square numbers in FIG 8 I also drew the graphs **S+1A, S+1B, S+1C** into FIG 8 (→ drawn in pink). These graphs are the next parallel graphs to the graphs SA, SB and SC in the negative rotation direction, and the numbers contained in these graphs are the square numbers +1. The same numbers are also contained in the two graphs marked with **S+1** in the Ulam Spiral (see image NS-18) and in the graph **S+1** on the Number Spiral. Note : this graph is not marked in image NS-8 , however it lies on the opposite side of graph S-1 in reference to graph S.

The main graphs of the second group of reference graphs are named with the letter **P** on the Ulam Spiral and Number Spiral (see FIG. NS-18 and / NS-8) and with the letters **PA, PB, PC** on the Square Root Spiral (graphs drawn in dark blue in FIG 8). → Note : letter **P** stays for " pronics"

The distribution of "Pronics-Numbers" (of the same type !) is similar to the distribution of the Square Numbers. It is as follows :

Number Spiral : Pronics Numbers are located on **1** spiral graph
Ulam Spiral : Pronics Numbers are located on **2** straight graphs
Square Root Spiral : Pronics Numbers are located on **3** spiral graphs

The difference in the number of graphs, on which pronics-numbers of the same type are located, has the same cause as already described for the square numbers , which is the difference in the tightness of the winding of the Ulam-, Number- and Square Root-Spiral.

Beside the "main" pronics-graphs PA, PB, PC (see FIG 8) I drew the next parallel graphs to these graphs into FIG 8. These Graphs are named P+1A, P+1B, P+1C and P-1A, P-1B, P-1C and P-2A, P-2B, P-2C. And their counterparts on the Ulam Spiral and on the Number Spiral are named P+1 as well as P-1 and P-2. → See FIG. NS-18 and / NS-8

Note : From these graphs only graph P-2 is marked on the Number Spiral (→ see NS-8). Because only the "pronics-graphs" P, P-2, P-6, P-12, P-20 etc. as well as the "S-graphs" S, S-1, S-4, S-9, S-16 etc. really contain "pronics-numbers".

Other P-graphs like P-4, P-8, P-14, P-16 etc. , which also lie in an even-number distance to graph P , also contain numbers which are composed by factors for which a certain rule applies. However these numbers are no real pronics !

Note: All P-graphs on the Number Spiral with an even-number-distance to graph P contain no Prime Numbers ! (→ see image NS-8)

It is different with the P-graphs which have an odd-number distance to graph P, like the graphs P-1, P-3, P-5 etc. or P+1, P+3, P+5 etc. (not marked in NS-8)

These graphs contain an above-average share of prime numbers ! For example graph P+1 and P-1 (see image NS-4 in chapter 9) or graph P+41 (see image NS-17). These graphs contain high shares in Prime Numbers !

Graph **P+41** is already well known in mathematics as the " **Euler-Polynomial**" . This graph which contains the number sequence 41, 43, 47, 53, 61, 71,…etc. and which contains a particular high share in Prime Numbers, is defined by the following quadratic polynomial : **$f(x) = x^2 + x + 41$** (see image NS 17)

On the Square Root Spiral the Euler Polynom divides into three spiral-graphs which I named **P+41A, P+41B** and **P+41C** (see FIG 8). The number sequences of these graphs are shown in **Table 7-B**. (→ page 35)

But let's first have a look to **Table 7-A** (page 34). This table shows the number sequences of the "reference-graphs" which I used to draw a comparison between the Ulam-Spiral, the Number-Spiral and the Square Root Spiral.

The lefthand side of Table 7-A shows the number sequences of the two main reference graphs S and P as they appear in image NS-8 of the Number Spiral.

Beside the number sequences S and P, the next three columns show the number sequences of the spiral graphs SA, SB, SC and PA, PB, PC which are the corresponding "S- and P-graphs" on the Square Root Spiral. (→ see FIG 8)

All above mentioned graphs are defined by "pronics-numbers". The factors of these pronics are shown in the yellow columns. In the number sequence of graph S the factors of the numbers increase by 0 and In the number sequence P the factors of the numbers increase by 1. It is the same with the factors in the numbers of the sequences SA, SB, SC and PA, PB, PC, however the factors of <u>successive</u> numbers in these sequences increase by 3.

By connecting the "pronics" on the spiral graphs PA, PB, PC with a continuous line, in the same order as they appear in the pronics-graph **P** (with an increase of 1 between the factors of <u>successive</u> numbers), a rotating triangular line pattern evolves (→ see red line pattern in FIG 8).
The angle between two successive lines of this triangular line pattern strives for

$\pi - 2$ for PA, PB, PC → ∞

Besides, a similar triangular line pattern would evolve if the square numbers in the graphs SA, SB, SC would be connected in the correct order by lines. And the angle in this triangular line pattern would strive for the same constant !

The factors of the pronics which I marked in red or blue in Table 7-A (see yellow columns), belong to two special number sequences which contain all existing prime numbers. I called these two special number sequences **SQ1** and **SQ2**
I had a closer look to these two important number sequences in another study which I intend to file with the arXiv – Archiv and which has the titel :

→ " The logic of the prime number distribution "

Number Sequence **SQ1** : 5, 11, 17, 23, 29, ... (→ numbers marked in blue)

Number Sequence **SQ2** : 1, 7, 13, 19, 25 , 31, ... (→ numbers marked in red)

Interesting are also the "sums of the digits" which occur in the number sequences of the reference graphs ! If we order the occurring sums of the digits according to their value, then the same sums of the digits sequences appear as already described for the prime number spiral graphs in Table 2.

Noticeable is here the "ordered" sums of the digits sequence which arises from the number sequence belonging to reference graph P (→ image NS-8).
The ordered sums of the digits sequence of graph P which is 2, 3, 6, 9, 11, 12, … shows the same periodicity (…..3, 3, 2, 1,….), of the differences between the numbers in this sequences, as the sums of the digits sequences of the prime-number-sequences with the 2.Differential = 20 shown in Table 2.

And the ordered sums of the digits sequences which arise from the number sequences belonging to reference graphs SA, SB, SC and PA, PB, PC have either a periodicity of 3 or 9.

Worth mentioning is also the periodic occurrence of groups of four numbers which are not divisible by 2, 3 or 5 in the number sequences SA and SB (marked in red).

On the righthand side of Table 7-A the number sequences S+1 and P-1 are shown. S+1 and P-1 are the two parallel graphs next to the reference graphs S and P in image NS-8 of the Number Spiral. These graphs are not marked in image NS-8 !

The next three columns on the right show the number sequences of the spiral graphs S+1A, S+1B, S+1C and P-1A, P-1B, P-1C which are the corresponding graphs on the Square Root Spiral. (→ see FIG 8).
Noticable is here that the number sequences P-1A, P-1B, P-1C are identical to the number sequences A3, B4, C5 shown in Table 6-A1, which are derived from the Prime Number Spiral Graphs A3, B4, C5 shown in FIG. 6-A !!

Worth mentioning are again the "ordered sums of the digits sequences", which arise from the number sequences S+1 and P-1 (Number Spiral), as well as from the number sequences S+1A, S+1B, S+1C and P-1A, P-1B, P-1C (Square Root Spiral).

The ordered sums of the digits sequences of graphs S+1 and P-1 are : 1, 5, 8, 10, 11, 14, 17, 19, 20, …… and 2, 5, 8, 10, 11, 14, 17, 19,…., which show the same periodicity (…..3, 3, 2, 1,….), of the differences between the numbers in this sequences, as reference graph P and the sums of the digits sequences of the prime-number-sequences with the 2. Differential = 20 shown in Table 2.

It would definitely be interesting to find the real reason for the often occurrence of this periodicity (...3, 3, 2, 1,…) which not only occurs in the Square Root Spiral but also in the Number Spiral !!

And the ordered sums of the digits sequences which arise from the number sequences belonging to the graphs S+1A, S+1B, S+1C and P-1A, P-1B, P-1C have either a periodicity of 3 or 9.

Noticable is here the periodic occurrence of groups of three numbers which are not divisible by 2, 3 or 5 in the number sequences S+1A, S+1B, S+1C as well as the periodic occurrence of groups of four such numbers in the number sequences P-1A, P-1B, P-1C.

I want to come back now to **Table 7-B** (→ page 35), which shows the number sequence of Graph **P+41** (see image NS 17 on page 32 – "The Number Spiral"), which is known as the " **Euler-Polynomial**" : $\mathbf{f(x) = x^2 + x + 41}$

As mentioned before, in the Square Root Spiral the graph **P+41** divides into three spiral-graphs which I named **P+41A, P+41B** and **P+41C** (see FIG 8). The number sequences of these graphs are shown in Table 7-B.

By the way, the spiral graphs and number sequences **P+41A, P+41B** and **P+41C are identical to** the prime number spiral graphs and corresponding number sequences **A17, B18** and **C 19 !!** Unfortunately I haven't carried out my analysis so far in FIG 6-A and Table 6-A1, otherwise these spiral graphs and number sequences would also have appeared here !!

The columns "prime factors" on the left of the number sequences P+41A, P+41B and P+41C show the prime factors of the first non-prime numbers in these sequences. In all three number sequences the smallest prime factor is 41.
Worth mentioning is also the fact, that the smallest number in the number sequences P+41A and P+41C is 41 , whereas the smallest number in the number sequence P+41B is 43.

Noticable is also the periodic occurrence of the prime factors in the non-prime numbers. For example prime factor 41 occurrs in the period 14/27 in number sequence P+41A and P+41B and in the period 27/14 in number sequence P+41C.

And prime factor 43 occurrs in the period 1/42 in number sequence P+41A and P+41C.

Important : As already described in Table 4 and 5A & 5B, the prime factors of the non-prime numbers of all quadratic polynomials, which lie on the Square Root Spiral, occur in clear defined periods !!
In **Table 7-C** (on the bottom of page 35) the quadratic polynomials of the number sequences P+41 (Number Spiral) and P+41A, P+41B, P+41C (Square Root Spiral) are shown.

Note: Different quadratic polynomials can be calculated for the above mentioned number-sequences. The resulting quadratic polynomial depends on the selection of the three numbers out of these number-sequences, which are used for the calculation of the quadratic polynomials.

I have calculated the first four quadratic polynomials for each number sequence (see Table 7-C).

Worth mentioning is here the following interdependence :

Referring to the general quadratic polynomial : $f(x) = \mathbf{ax^2 + bx + c}$

the following pattern evolves in **Table 7-C** for the coefficients **a, b** and **c** :

- **a** → is always equivalent to the **2.Differential** of the Spiral-Graph **divided by two** (which is 18/2 = 9 for the quadratic polynomials P+41A, P+41B and P+41C)
- **b** → this coefficient **increases by the value of the 2.Differential** from column to column shown in Table 7-C
- **c** → this coefficient is always **equal to the number which comes before the three numbers, which were chosen to calculate the quadratic polynomial** in the number sequence belonging to this quadratic polynomial.
  (Note : this rule doesn's apply to the first quadratic polynomial which was calculated with the first three numbers of the number-sequence, because of course there is no number before the first three numbers !)

By the way, the same pattern exists in the Tables 6-A2 to 6-C2, which show the first four calculated quadratic polynomials of the Prime Number Spiral Graphs shown in FIG 6-A to 6-C !!

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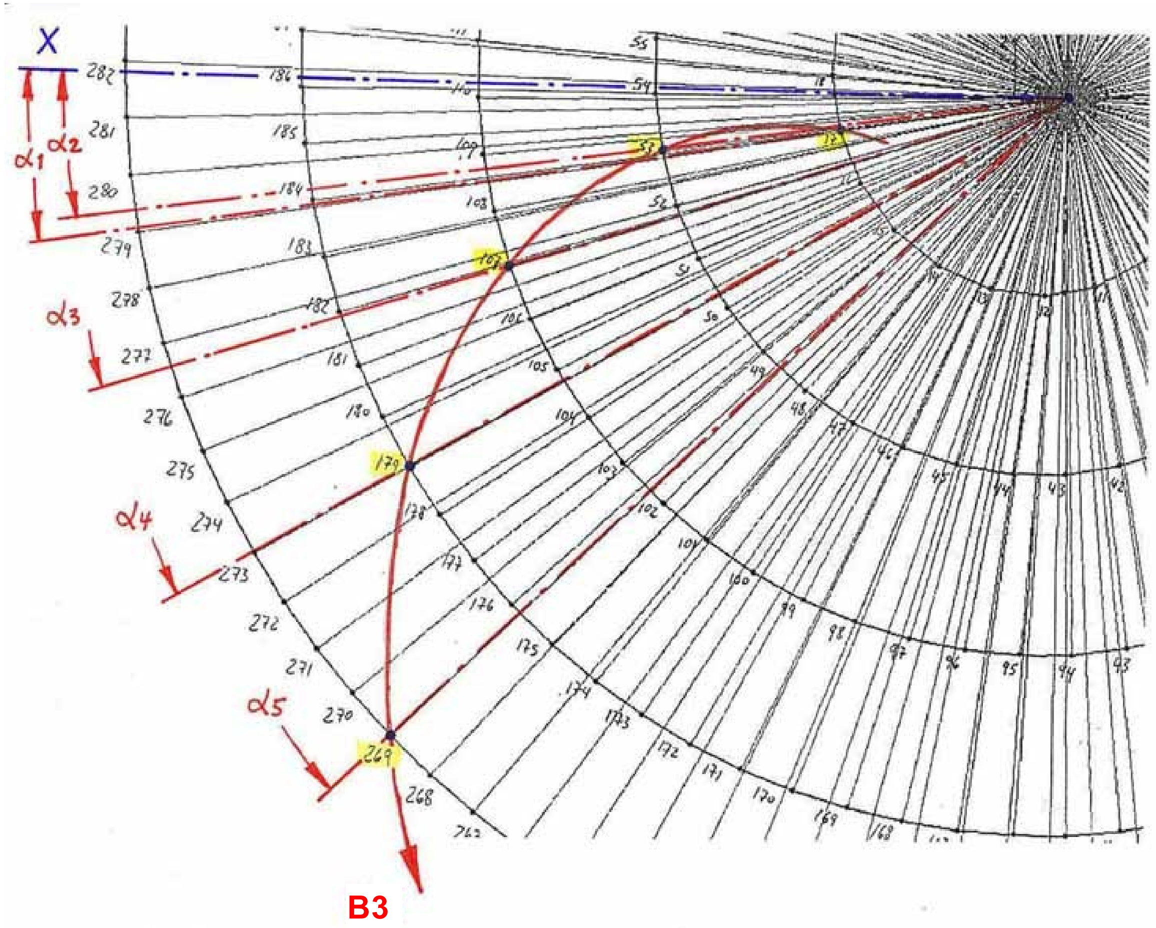
Appendix :

This image shows exemplary some polar coordinates of the "Prime Number Spiral Graph" **B3** → see **FIG. 6-A**



Prime Number Spiral Graph B3 :

Root	α	Angle to the X-Axis
$\sqrt{17}$	1	8,84957988°
$\sqrt{53}$	2	8,08226071°
$\sqrt{107}$	3	17,36077355°
$\sqrt{179}$	4	29,78532935°
$\sqrt{269}$	5	43,60654265°

Harry K. Hahn / 1.5.2006

Table 6-B1 : “Prime Number Sequences” derived from the graphs schown in the “Prime Number Spiral Systems” **N20-D** to **N20-I** and **P20-D** to **P20-I** → see **FIG. 6-B**

N20 - D1

SD	D1	D1'	D1"
7	7		
9	27	20	
13	67	40	20
10	127	60	20
9	207	80	20
10	307	100	20
13	427	120	20
18	567	140	20
16	727	160	20
16	907	180	20
9	1107	200	20
13	1327	220	20
19	1567	240	20
18	1827	260	20
10	2107	280	20
13	2407	300	20
18	2727	320	20
16	3067	340	20
16	3427	360	20
18	3807	380	20
13	4207	400	20
19	4627	420	20
18	5067	440	20
19	5527	460	20
13	6007	480	20
18	6507	500	20
16	7027	520	20

N20 - D2

SD	D2	D2'	D2"
5	5		
7	25	20	
13	65	40	20
8	125	60	20
7	205	80	20
8	305	100	20
11	425	120	20
16	565	140	20
14	725	160	20
14	905	180	20
7	1105	200	20
12	1325	220	20
17	1565	240	20
16	1825	260	20
8	2105	280	20
11	2405	300	20
16	2725	320	20
14	3065	340	20
14	3425	360	20
16	3805	380	20
11	4205	400	20
17	4625	420	20
16	5065	440	20
17	5525	460	20
11	6005	480	20
16	6505	500	20
14	7025	520	20

- 4 →

- 4 →

N20 - D3

SD	D3	D3'	D3"
1	1		
3	21	20	
7	61	40	20
4	121	60	20
3	201	80	20
4	301	100	20
7	421	120	20
12	561	140	20
10	721	160	20
10	901	180	20
3	1101	200	20
7	1321	220	20
13	1561	240	20
12	1821	260	20
4	2101	280	20
7	2401	300	20
12	2721	320	20
10	3061	340	20
10	3421	360	20
12	3801	380	20
7	4201	400	20
13	4621	420	20
12	5061	440	20
13	5521	460	20
7	6001	480	20
12	6501	500	20
10	7021	520	20

- 2 →

- 2 →

P20 - D1

SD	D1	D1'	D1"
2	11		
4	31	20	
8	71	40	20
5	131	60	20
4	211	80	20
5	311	100	20
8	431	120	20
13	571	140	20
11	731	160	20
11	911	180	20
4	1111	200	20
8	1331	220	20
14	1571	240	20
13	1831	260	20
5	2111	280	20
8	2411	300	20
13	2731	320	20
11	3071	340	20
11	3431	360	20
13	3811	380	20
8	4211	400	20
14	4631	420	20
13	5071	440	20
14	5531	460	20
8	6011	480	20
13	6511	500	20
11	7031	520	20

P20 - D2

SD	D2	D2'	D2"
4	13		
6	33	20	
10	73	40	20
7	133	60	20
6	213	80	20
7	313	100	20
10	433	120	20
15	573	140	20
13	733	160	20
13	913	180	20
6	1113	200	20
10	1333	220	20
16	1573	240	20
15	1833	260	20
7	2113	280	20
10	2413	300	20
15	2733	320	20
13	3073	340	20
13	3433	360	20
15	3813	380	20
10	4213	400	20
16	4633	420	20
15	5073	440	20
16	5533	460	20
10	6013	480	20
15	6513	500	20
13	7033	520	20

+ 4 →

+ 4 →

P20 - D3

SD	D3	D3'	D3"
8	17		
10	37	20	
14	77	40	20
11	137	60	20
10	217	80	20
11	317	100	20
14	437	120	20
19	577	140	20
17	737	160	20
17	917	180	20
10	1117	200	20
14	1337	220	20
20	1577	240	20
19	1837	260	20
11	2117	280	20
14	2417	300	20
19	2737	320	20
17	3077	340	20
17	3437	360	20
19	3817	380	20
14	4217	400	20
20	4637	420	20
19	5077	440	20
20	5537	460	20
14	6017	480	20
20	6517	500	20
17	7037	520	20

+ 2 →

+ 2 →

N20 - E1

SD	E1	E1'	E1"
7	7		
4	31	24	
12	75	44	20
13	139	64	20
7	223	84	20
12	327	104	20
10	451	124	20
19	595	144	20
21	759	164	20
16	943	184	20
13	1147	204	20
12	1371	224	20
13	1615	244	20
25	1879	264	20
12	2163	284	20
19	2467	304	20
19	2791	324	20
12	3135	344	20
25	3499	364	20
22	3883	384	20
21	4287	404	20
13	4711	424	20
16	5155	444	20
21	5619	464	20
10	6103	484	20
19	6607	504	20
12	7131	524	20

N20 - E2

SD	E2	E2'	E2"
5	5		
11	29	24	
10	73	44	20
11	137	64	20
5	221	84	20
10	325	104	20
17	449	124	20
17	593	144	20
19	757	164	20
14	941	184	20
11	1145	204	20
19	1369	224	20
11	1613	244	20
23	1877	264	20
10	2161	284	20
17	2465	304	20
26	2789	324	20
10	3133	344	20
23	3497	364	20
20	3881	384	20
19	4285	404	20
20	4709	424	20
14	5153	444	20
19	5617	464	20
8	6101	484	20
17	6605	504	20
19	7129	524	20

- 4 →

- 4 →

N20 - E3

SD	E3	E3'	E3"
1	1		
7	25	24	
15	69	44	20
7	133	64	20
10	217	84	20
6	321	104	20
13	445	124	20
22	589	144	20
15	753	164	20
19	937	184	20
7	1141	204	20
15	1365	224	20
16	1609	244	20
19	1873	264	20
15	2157	284	20
13	2461	304	20
22	2785	324	20
15	3129	344	20
19	3493	364	20
25	3877	384	20
15	4281	404	20
16	4705	424	20
19	5149	444	20
15	5613	464	20
22	6097	484	20
13	6601	504	20
15	7125	524	20

- 2 →

- 2 →

P20 - E1

SD	E1	E1'	E1"
2	11		
8	35	24	
16	79	44	20
8	143	64	20
11	227	84	20
7	331	104	20
14	455	124	20
23	599	144	20
16	763	164	20
20	947	184	20
8	1151	204	20
16	1375	224	20
17	1619	244	20
20	1883	264	20
16	2167	284	20
14	2471	304	20
23	2795	324	20
16	3139	344	20
11	3503	364	20
26	3887	384	20
16	4291	404	20
17	4715	424	20
20	5159	444	20
16	5623	464	20
14	6107	484	20
14	6611	504	20
16	7135	524	20

P20 - E2

SD	E2	E2'	E2"
4	13		
10	37	24	
9	81	44	20
10	145	64	20
13	229	84	20
9	333	104	20
16	457	124	20
7	601	144	20
18	765	164	20
22	949	184	20
10	1153	204	20
18	1377	224	20
10	1621	244	20
22	1885	264	20
18	2169	284	20
16	2473	304	20
25	2797	324	20
9	3141	344	20
13	3505	364	20
28	3889	384	20
18	4293	404	20
19	4717	424	20
13	5161	444	20
18	5625	464	20
16	6109	484	20
16	6613	504	20
18	7137	524	20

+ 4 →

+ 4 →

P20 - E3

SD	E3	E3'	E3"
8	17		
5	41	24	
13	85	44	20
14	149	64	20
8	233	84	20
13	337	104	20
11	461	124	20
11	605	144	20
22	769	164	20
17	953	184	20
14	1157	204	20
13	1381	224	20
14	1625	244	20
26	1889	264	20
13	2173	284	20
20	2477	304	20
11	2801	324	20
13	3145	344	20
17	3509	364	20
23	3893	384	20
22	4297	404	20
14	4721	424	20
17	5165	444	20
22	5629	464	20
11	6113	484	20
20	6617	504	20
13	7141	524	20

+ 2 →

+ 2 →

N20 - F1

SD	F1	F1'	F1"
7	7		
6	33	26	
16	79	46	20
10	145	66	20
6	231	86	20
13	337	106	20
13	463	126	20
15	609	146	20
19	775	166	20
16	961	186	20
15	1167	206	20
16	1393	226	20
19	1639	246	20
15	1905	266	20
14	2191	286	20
22	2497	306	20
15	2823	326	20
19	3169	346	20
16	3535	366	20
15	3921	386	20
16	4327	406	20
19	4753	426	20
24	5199	446	20
22	5665	466	20
13	6151	486	20
24	6657	506	20
19	7183	526	20

N20 - F2

SD	F2	F2'	F2"
5	5		
4	31	26	
14	77	46	20
8	143	66	20
13	229	86	20
11	335	106	20
11	461	126	20
13	607	146	20
17	773	166	20
23	959	186	20
13	1165	206	20
14	1391	226	20
17	1637	246	20
13	1903	266	20
20	2189	286	20
20	2495	306	20
13	2821	326	20
17	3167	346	20
14	3533	366	20
22	3919	386	20
14	4325	406	20
17	4751	426	20
22	5197	446	20
20	5663	466	20
11	6149	486	20
22	6655	506	20
17	7181	526	20

- 4 →

- 4 →

N20 - F3

SD	F3	F3'	F3"
1	1		
9	27	26	
10	73	46	20
13	139	66	20
9	225	86	20
7	331	106	20
16	457	126	20
9	603	146	20
22	769	166	20
19	955	186	20
9	1161	206	20
19	1387	226	20
13	1633	246	20
27	1899	266	20
16	2185	286	20
16	2491	306	20
18	2817	326	20
13	3163	346	20
19	3529	366	20
18	3915	386	20
10	4321	406	20
22	4747	426	20
18	5193	446	20
28	5659	466	20
16	6145	486	20
18	6651	506	20
22	7177	526	20

- 2 →

- 2 →

P20 - F1

SD	F1	F1'	F1"
2	11		
10	37	26	
11	83	46	20
14	149	66	20
10	235	86	20
8	341	106	20
17	467	126	20
10	613	146	20
23	779	166	20
20	965	186	20
10	1171	206	20
20	1397	226	20
14	1643	246	20
19	1909	266	20
17	2195	286	20
8	2501	306	20
19	2827	326	20
14	3173	346	20
20	3539	366	20
19	3925	386	20
11	4331	406	20
23	4757	426	20
10	5203	446	20
26	5669	466	20
17	6155	486	20
19	6661	506	20
23	7187	526	20

P20 - F2

SD	F2	F2'	F2"
4	13		
12	39	26	
13	85	46	20
7	151	66	20
12	237	86	20
10	343	106	20
19	469	126	20
12	615	146	20
16	781	166	20
22	967	186	20
12	1173	206	20
22	1399	226	20
16	1645	246	20
12	1911	266	20
19	2197	286	20
10	2503	306	20
21	2829	326	20
16	3175	346	20
13	3541	366	20
21	3927	386	20
13	4333	406	20
25	4759	426	20
12	5205	446	20
19	5671	466	20
19	6157	486	20
21	6663	506	20
25	7189	526	20

+ 4 →

+ 4 →

P20 - F3

SD	F3	F3'	F3"
8	17		
7	43	26	
17	89	46	20
11	155	66	20
7	241	86	20
14	347	106	20
14	473	126	20
16	619	146	20
20	785	166	20
17	971	186	20
16	1177	206	20
8	1403	226	20
20	1649	246	20
16	1915	266	20
5	2201	286	20
14	2507	306	20
16	2833	326	20
20	3179	346	20
17	3545	366	20
16	3931	386	20
17	4337	406	20
20	4763	426	20
16	5209	446	20
23	5675	466	20
14	6161	486	20
25	6667	506	20
20	7193	526	20

+ 2 →

+ 2 →

N20 - G1

SD	G1	G1'	G1"
10	19		
13	49	30	
18	99	50	20
16	169	70	20
16	259	90	20
18	369	110	20
22	499	130	20
19	649	150	20
18	819	170	20
10	1009	190	20
13	1219	210	20
18	1449	230	20
25	1699	250	20
25	1969	270	20
18	2259	290	20
22	2569	310	20
28	2899	330	20
18	3249	350	20
19	3619	370	20
13	4009	390	20
18	4419	410	20
25	4849	430	20
25	5299	450	20
27	5769	470	20
22	6259	490	20
28	6769	510	20
27	7299	530	20

N20 - G2

SD	G2	G2'	G2"
8	17		
11	47	30	
16	97	50	20
14	167	70	20
14	257	90	20
16	367	110	20
20	497	130	20
17	647	150	20
16	817	170	20
8	1007	190	20
11	1217	210	20
16	1447	230	20
23	1697	250	20
23	1967	270	20
16	2257	290	20
20	2567	310	20
26	2897	330	20
16	3247	350	20
17	3617	370	20
11	4007	390	20
16	4417	410	20
23	4847	430	20
23	5297	450	20
25	5767	470	20
20	6257	490	20
26	6767	510	20
25	7297	530	20

- 4 →

- 4 →

N20 - G3

SD	G3	G3'	G3"
4	13		
7	43	30	
12	93	50	20
10	163	70	20
10	253	90	20
12	363	110	20
16	493	130	20
13	643	150	20
12	813	170	20
4	1003	190	20
7	1213	210	20
12	1443	230	20
19	1693	250	20
19	1963	270	20
12	2253	290	20
16	2563	310	20
22	2893	330	20
12	3243	350	20
13	3613	370	20
7	4003	390	20
12	4413	410	20
19	4843	430	20
19	5293	450	20
21	5763	470	20
16	6253	490	20
22	6763	510	20
21	7293	530	20

- 2 →

- 2 →

P20 - G1

SD	G1	G1'	G1"
5	23		
8	53	30	
4	103	50	20
11	173	70	20
11	263	90	20
13	373	110	20
8	503	130	20
14	653	150	20
13	823	170	20
5	1013	190	20
8	1223	210	20
13	1453	230	20
11	1703	250	20
20	1973	270	20
13	2263	290	20
17	2573	310	20
14	2903	330	20
13	3253	350	20
14	3623	370	20
8	4013	390	20
13	4423	410	20
20	4853	430	20
11	5303	450	20
22	5773	470	20
17	6263	490	20
23	6773	510	20
13	7303	530	20

P20 - G2

SD	G2	G2'	G2"
	25		
	55	30	
	105	50	20
	175	70	20
	265	90	20
	375	110	20
	505	130	20
	655	150	20
	825	170	20
	1015	190	20
	1225	210	20
	1455	230	20
	1705	250	20
	1975	270	20
	2265	290	20
	2575	310	20
	2905	330	20
	3255	350	20
	3625	370	20
	4015	390	20
	4425	410	20
	4855	430	20
	5305	450	20
	5775	470	20
	6265	490	20
	6775	510	20
	7305	530	20

+ 4 →

+ 4 →

P20 - G3

SD	G3	G3'	G3"
11	29		
14	59	30	
10	109	50	20
17	179	70	20
17	269	90	20
19	379	110	20
14	509	130	20
20	659	150	20
19	829	170	20
11	1019	190	20
14	1229	210	20
19	1459	230	20
17	1709	250	20
26	1979	270	20
19	2269	290	20
23	2579	310	20
20	2909	330	20
19	3259	350	20
20	3629	370	20
14	4019	390	20
19	4429	410	20
26	4859	430	20
17	5309	450	20
28	5779	470	20
23	6269	490	20
29	6779	510	20
19	7309	530	20

+ 2 →

+ 2 →

N20 - H1

SD	H1	H1'	H1"
3	21		
10	55	34	
10	109	54	20
12	183	74	20
16	277	94	20
13	391	114	20
12	525	134	20
22	679	154	20
16	853	174	20
12	1047	194	20
10	1261	214	20
19	1495	234	20
21	1749	254	20
7	2023	274	20
13	2317	294	20
12	2631	314	20
22	2965	334	20
16	3319	354	20
21	3693	374	20
19	4087	394	20
10	4501	414	20
21	4935	434	20
25	5389	454	20
22	5863	474	20
21	6357	494	20
22	6871	514	20
16	7405	534	20

N20 - H2

SD	H2	H2'	H2"
10	19		
8	53	34	
8	107	54	20
10	181	74	20
14	275	94	20
20	389	114	20
10	523	134	20
20	677	154	20
14	851	174	20
10	1045	194	20
17	1259	214	20
17	1493	234	20
19	1747	254	20
5	2021	274	20
11	2315	294	20
19	2629	314	20
20	2963	334	20
14	3317	354	20
19	3691	374	20
17	4085	394	20
26	4499	414	20
19	4933	434	20
23	5387	454	20
20	5861	474	20
19	6355	494	20
29	6869	514	20
14	7403	534	20

- 4 →

- 4 →

N20 - H3

SD	H3	H3'	H3"
6	15		
13	49	34	
4	103	54	20
15	177	74	20
10	271	94	20
16	385	114	20
15	519	134	20
16	673	154	20
19	847	174	20
6	1041	194	20
13	1255	214	20
22	1489	234	20
15	1743	254	20
10	2017	274	20
7	2311	294	20
15	2625	314	20
25	2959	334	20
10	3313	354	20
24	3687	374	20
13	4081	394	20
22	4495	414	20
24	4929	434	20
19	5383	454	20
25	5857	474	20
15	6351	494	20
25	6865	514	20
28	7399	534	20

- 2 →

- 2 →

P20 - H1

SD	H1	H1'	H1"
7	25		
14	59	34	
5	113	54	20
16	187	74	20
11	281	94	20
17	395	114	20
16	529	134	20
17	683	154	20
20	857	174	20
7	1051	194	20
14	1265	214	20
23	1499	234	20
16	1753	254	20
11	2027	274	20
8	2321	294	20
16	2635	314	20
26	2969	334	20
11	3323	354	20
25	3697	374	20
14	4091	394	20
14	4505	414	20
25	4939	434	20
20	5393	454	20
26	5867	474	20
16	6361	494	20
26	6875	514	20
20	7409	534	20

P20 - H2

SD	H2	H2'	H2"
9	27		
7	61	34	
7	115	54	20
18	189	74	20
13	283	94	20
19	397	114	20
9	531	134	20
19	685	154	20
22	859	174	20
9	1053	194	20
16	1267	214	20
7	1501	234	20
18	1755	254	20
13	2029	274	20
10	2323	294	20
18	2637	314	20
19	2971	334	20
13	3325	354	20
27	3699	374	20
16	4093	394	20
16	4507	414	20
18	4941	434	20
22	5395	454	20
28	5869	474	20
18	6363	494	20
28	6877	514	20
13	7411	534	20

+ 4 →

+ 4 →

P20 - H3

SD	H3	H3'	H3"
4	31		
11	65	34	
11	119	54	20
13	193	74	20
17	287	94	20
5	401	114	20
13	535	134	20
23	689	154	20
17	863	174	20
13	1057	194	20
11	1271	214	20
11	1505	234	20
22	1759	254	20
8	2033	274	20
14	2327	294	20
13	2641	314	20
23	2975	334	20
17	3329	354	20
13	3703	374	20
20	4097	394	20
11	4511	414	20
22	4945	434	20
26	5399	454	20
23	5873	474	20
22	6367	494	20
23	6881	514	20
17	7415	534	20

+ 2 →

+ 2 →

N20 - I1

SD	I1	I1'	I1"
7	25		
7	61	36	
9	117	56	20
13	193	76	20
19	289	96	20
9	405	116	20
10	541	136	20
22	697	156	20
18	873	176	20
16	1069	196	20
16	1285	216	20
9	1521	236	20
22	1777	256	20
10	2053	276	20
18	2349	296	20
19	2665	316	20
4	3001	336	20
18	3357	356	20
16	3733	376	20
16	4129	396	20
18	4545	416	20
22	4981	436	20
19	5437	456	20
18	5913	476	20
19	6409	496	20
22	6925	516	20
18	7461	536	20

N20 - I2

SD	I2	I2'	I2"
5	23		
14	59	36	
7	115	56	20
11	191	76	20
17	287	96	20
7	403	116	20
17	539	136	20
20	695	156	20
16	871	176	20
14	1067	196	20
14	1283	216	20
16	1519	236	20
20	1775	256	20
8	2051	276	20
16	2347	296	20
17	2663	316	20
29	2999	336	20
16	3355	356	20
14	3731	376	20
14	4127	396	20
16	4543	416	20
29	4979	436	20
17	5435	456	20
16	5911	476	20
17	6407	496	20
20	6923	516	20
25	7459	536	20

- 4 →

- 4 →

N20 - I3

SD	I3	I3'	I3"
10	19		
10	55	36	
3	111	56	20
16	187	76	20
13	283	96	20
21	399	116	20
13	535	136	20
16	691	156	20
21	867	176	20
10	1063	196	20
19	1279	216	20
12	1515	236	20
16	1771	256	20
13	2047	276	20
12	2343	296	20
22	2659	316	20
25	2995	336	20
12	3351	356	20
19	3727	376	20
10	4123	396	20
21	4539	416	20
25	4975	436	20
13	5431	456	20
21	5907	476	20
13	6403	496	20
25	6919	516	20
21	7455	536	20

- 2 →

- 2 →

P20 - I1

SD	I1	I1'	I1"
11	29		
11	65	36	
4	121	56	20
17	197	76	20
14	293	96	20
13	409	116	20
14	545	136	20
8	701	156	20
22	877	176	20
11	1073	196	20
20	1289	216	20
13	1525	236	20
17	1781	256	20
14	2057	276	20
13	2353	296	20
23	2669	316	20
8	3005	336	20
13	3361	356	20
20	3737	376	20
11	4133	396	20
22	4549	416	20
26	4985	436	20
14	5441	456	20
22	5917	476	20
14	6413	496	20
26	6929	516	20
22	7465	536	20

P20 - I2

SD	I2	I2'	I2"
4	31		
13	67	36	
6	123	56	20
19	199	76	20
16	295	96	20
6	411	116	20
16	547	136	20
10	703	156	20
24	879	176	20
13	1075	196	20
13	1291	216	20
15	1527	236	20
19	1783	256	20
16	2059	276	20
15	2355	296	20
16	2671	316	20
10	3007	336	20
15	3363	356	20
22	3739	376	20
13	4135	396	20
15	4551	416	20
28	4987	436	20
16	5443	456	20
24	5919	476	20
16	6415	496	20
19	6931	516	20
24	7467	536	20

+ 4 →

+ 4 →

P20 - I3

SD	I3	I3'	I3"
8	35		
8	71	36	
10	127	56	20
5	203	76	20
20	299	96	20
10	415	116	20
11	551	136	20
14	707	156	20
19	883	176	20
17	1079	196	20
17	1295	216	20
10	1531	236	20
23	1787	256	20
11	2063	276	20
19	2359	296	20
20	2675	316	20
5	3011	336	20
19	3367	356	20
17	3743	376	20
17	4139	396	20
19	4555	416	20
23	4991	436	20
20	5447	456	20
19	5923	476	20
20	6419	496	20
23	6935	516	20
19	7471	536	20

+ 2 →

+ 2 →

Table 6-C1 : “Prime Number Sequences” derived from the graphs schown in the “Prime Number Spiral Systems” **N22-J** to **N22-K** → see **FIG. 6-C**

SD	N22 - J1 J1	J1'	J1"	SD	N22 - J2 J2	J2'	J2"	SD	N22 - J3 J3	J3'	J3"
4	31			11	29			7	25		
15	69	38		13	67	38		9	63	38	
12	129	60	22	10	127	60	22	6	123	60	22
4	211	82	22	11	209	82	22	7	205	82	22
9	315	104	22	7	313	104	22	12	309	104	22
9	441	126	22	16	439	126	22	12	435	126	22
22	589	148	22	20	587	148	22	16	583	148	22
21	759	170	22	19	757	170	22	15	753	170	22
15	951	192	22	22	949	192	22	18	945	192	22
13	1165	214	22	11	1163	214	22	16	1159	214	22
6	1401	236	22	22	1399	236	22	18	1395	236	22
21	1659	258	22	19	1657	258	22	15	1653	258	22
22	1939	280	22	20	1937	280	22	16	1933	280	22
9	2241	302	22	16	2239	302	22	12	2235	302	22
18	2565	324	22	14	2563	324	22	21	2559	324	22
13	2911	346	22	20	2909	346	22	16	2905	346	22
21	3279	368	22	19	3277	368	22	15	3273	368	22
24	3669	390	22	22	3667	390	22	18	3663	390	22
13	4081	412	22	20	4079	412	22	16	4075	412	22
15	4515	434	22	13	4513	434	22	18	4509	434	22
21	4971	456	22	28	4969	456	22	24	4965	456	22
22	5449	478	22	20	5447	478	22	16	5443	478	22
27	5949	500	22	25	5947	500	22	21	5943	500	22
18	6471	522	22	25	6469	522	22	21	6465	522	22
13	7015	544	22	11	7013	544	22	16	7009	544	22
21	7581	566	22	28	7579	566	22	24	7575	566	22
24	8169	588	22	22	8167	588	22	18	8163	588	22
31	8779	610	22	29	8777	610	22	29	8773	610	22

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SD	N22 - K1 K1	K1'	K1"	SD	N22 - K2 K2	K2'	K2"	SD	N22 - K3 K3	K3'	K3"
7	25			5	23			10	19		
8	61	36		14	59	36		10	55	36	
11	119	58	22	9	117	58	22	5	113	58	22
19	199	80	22	17	197	80	22	13	193	80	22
4	301	102	22	20	299	102	22	16	295	102	22
11	425	124	22	9	423	124	22	14	419	124	22
13	571	146	22	20	569	146	22	16	565	146	22
19	739	168	22	17	737	168	22	13	733	168	22
20	929	190	22	18	927	190	22	14	923	190	22
7	1141	212	22	14	1139	212	22	10	1135	212	22
16	1375	234	22	14	1373	234	22	19	1369	234	22
11	1631	256	22	18	1629	256	22	14	1625	256	22
19	1909	278	22	17	1907	278	22	13	1903	278	22
13	2209	300	22	11	2207	300	22	7	2203	300	22
11	2531	322	22	18	2529	322	22	14	2525	322	22
22	2875	344	22	20	2873	344	22	25	2869	344	22
10	3241	366	22	17	3239	366	22	13	3235	366	22
20	3629	388	22	18	3627	388	22	14	3623	388	22
16	4039	410	22	14	4037	410	22	10	4033	410	22
16	4471	432	22	23	4469	432	22	17	4465	432	22
20	4925	454	22	18	4923	454	22	23	4919	454	22
10	5401	476	22	26	5399	476	22	22	5395	476	22
31	5899	498	22	29	5897	498	22	25	5893	498	22
20	6419	520	22	18	6417	520	22	14	6413	520	22
22	6961	542	22	29	6959	542	22	25	6955	542	22
19	7525	564	22	17	7523	564	22	22	7519	564	22
11	8111	586	22	18	8109	586	22	14	8105	586	22
25	8719	608	22	23	8717	608	22	19	8713	608	22

- 4 → - 2 →

SD	N22 - L1 L1	L1'	L1"	SD	N22 - L2 L2	L2'	L2"	SD	N22 - L3 L3	L3'	L3"
7	25			5	23			10	19		
14	59	34		12	57	34		8	53	34	
7	115	56	22	5	113	56	22	10	109	56	22
13	193	78	22	11	191	78	22	16	187	78	22
14	293	100	22	12	291	100	22	17	287	100	22
10	415	122	22	8	413	122	22	13	409	122	22
19	559	144	22	17	557	144	22	13	553	144	22
14	725	166	22	12	723	166	22	17	719	166	22
13	913	188	22	11	911	188	22	16	907	188	22
7	1123	210	22	5	1121	210	22	10	1117	210	22
14	1355	232	22	12	1353	232	22	17	1349	232	22
16	1609	254	22	14	1607	254	22	10	1603	254	22
22	1885	276	22	20	1883	276	22	25	1879	276	22
14	2183	298	22	12	2181	298	22	17	2177	298	22
10	2503	320	22	8	2501	320	22	22	2497	320	22
19	2845	342	22	17	2843	342	22	22	2839	342	22
14	3209	364	22	12	3207	364	22	8	3203	364	22
22	3595	386	22	20	3593	386	22	25	3589	386	22
7	4003	408	22	5	4001	408	22	28	3997	408	22
14	4433	430	22	12	4431	430	22	17	4427	430	22
25	4885	452	22	23	4883	452	22	28	4879	452	22
22	5359	474	22	20	5357	474	22	16	5353	474	22
23	5855	496	22	21	5853	496	22	26	5849	496	22
19	6373	518	22	17	6371	518	22	22	6367	518	22
19	6913	540	22	17	6911	540	22	22	6907	540	22
23	7475	562	22	21	7473	562	22	26	7469	562	22
22	8059	584	22	20	8057	584	22	16	8053	584	22
25	8665	606	22	23	8663	606	22	28	8659	606	22

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SD	N22 - M1 M1	M1'	M1"	SD	N22 - M2 M2	M2'	M2"	SD	N22 - M3 M3	M3'	M3"
5	23			3	21			8	17		
10	55	32		8	53	32		13	49	32	
10	109	54	22	8	107	54	22	4	103	54	22
14	185	76	22	12	183	76	22	17	179	76	22
13	283	98	22	11	281	98	22	16	277	98	22
7	403	120	22	5	401	120	22	19	397	120	22
14	545	142	22	12	543	142	22	17	539	142	22
16	709	164	22	14	707	164	22	10	703	164	22
22	895	186	22	20	893	186	22	25	889	186	22
5	1103	208	22	3	1101	208	22	17	1097	208	22
10	1333	230	22	8	1331	230	22	13	1327	230	22
19	1585	252	22	17	1583	252	22	22	1579	252	22
23	1859	274	22	21	1857	274	22	17	1853	274	22
13	2155	296	22	11	2153	296	22	16	2149	296	22
16	2473	318	22	14	2471	318	22	19	2467	318	22
14	2813	340	22	12	2811	340	22	17	2807	340	22
16	3175	362	22	14	3173	362	22	19	3169	362	22
22	3559	384	22	20	3557	384	22	16	3553	384	22
23	3965	406	22	21	3963	406	22	26	3959	406	22
19	4393	428	22	17	4391	428	22	22	4387	428	22
19	4843	450	22	17	4841	450	22	22	4837	450	22
14	5315	472	22	12	5313	472	22	17	5309	472	22
22	5809	494	22	20	5807	494	22	16	5803	494	22
16	6325	516	22	14	6323	516	22	19	6319	516	22
23	6863	538	22	21	6861	538	22	26	6857	538	22
16	7423	560	22	14	7421	560	22	19	7417	560	22
13	8005	582	22	11	8003	582	22	34	7999	582	22
23	8609	604	22	21	8607	604	22	17	8603	604	22

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SD	N22 - N1 N1	N1'	N1"	SD	N22 - N2 N2	N2'	N2"	SD	N22 - N3 N3	N3'	N3"
10	19			8	17			4	13		
13	49	30		11	47	30		7	43	30	
2	101	52	22	18	99	52	22	14	95	52	22
13	175	74	22	11	173	74	22	16	169	74	22
10	271	96	22	17	269	96	22	13	265	96	22
20	389	118	22	18	387	118	22	14	383	118	22
16	529	140	22	14	527	140	22	10	523	140	22
16	691	162	22	23	689	162	22	19	685	162	22
20	875	184	22	18	873	184	22	23	869	184	22
10	1081	206	22	17	1079	206	22	13	1075	206	22
13	1309	228	22	11	1307	228	22	7	1303	228	22
20	1559	250	22	18	1557	250	22	14	1553	250	22
13	1831	272	22	20	1829	272	22	16	1825	272	22
10	2125	294	22	8	2123	294	22	13	2119	294	22
11	2441	316	22	18	2439	316	22	14	2435	316	22
25	2779	338	22	23	2777	338	22	19	2773	338	22
16	3139	360	22	14	3137	360	22	10	3133	360	22
11	3521	382	22	18	3519	382	22	14	3515	382	22
19	3925	404	22	17	3923	404	22	22	3919	404	22
13	4351	426	22	20	4349	426	22	16	4345	426	22
29	4799	448	22	27	4797	448	22	23	4793	448	22
22	5269	470	22	20	5267	470	22	16	5263	470	22
19	5761	492	22	26	5759	492	22	22	5755	492	22
20	6275	514	22	18	6273	514	22	23	6269	514	22
16	6811	536	22	23	6809	536	22	19	6805	536	22
25	7369	558	22	23	7367	558	22	19	7363	558	22
29	7949	580	22	27	7947	580	22	23	7943	580	22

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SD	N22 - O1 O1	O1'	O1"	SD	N22 - O2 O2	O2'	O2"	SD	N22 - O3 O3	O3'	O3"
7	25			5	23			10	19		
8	53	28		6	51	28		11	47	28	
4	103	50	22	2	101	50	22	16	97	50	22
13	175	72	22	11	173	72	22	16	169	72	22
17	269	94	22	15	267	94	22	11	263	94	22
16	385	116	22	14	383	116	22	19	379	116	22
10	523	138	22	8	521	138	22	13	517	138	22
17	683	160	22	15	681	160	22	20	677	160	22
19	865	182	22	17	863	182	22	22	859	182	22
16	1069	204	22	14	1067	204	22	10	1063	204	22
17	1295	226	22	15	1293	226	22	20	1289	226	22
13	1543	248	22	11	1541	248	22	16	1537	248	22
13	1813	270	22	11	1811	270	22	16	1807	270	22
8	2105	292	22	6	2103	292	22	20	2099	292	22
16	2419	314	22	14	2417	314	22	10	2413	314	22
19	2755	336	22	17	2753	336	22	22	2749	336	22
8	3113	358	22	6	3111	358	22	11	3107	358	22
19	3493	380	22	17	3491	380	22	22	3487	380	22
25	3895	402	22	23	3893	402	22	28	3889	402	22
17	4319	424	22	15	4317	424	22	11	4313	424	22
22	4765	446	22	20	4763	446	22	25	4759	446	22
13	5233	468	22	11	5231	468	22	16	5227	468	22
17	5723	490	22	15	5721	490	22	20	5717	490	22
16	6235	512	22	14	6233	512	22	22	6229	512	22
28	6769	534	22	26	6767	534	22	22	6763	534	22
17	7325	556	22	15	7323	556	22	20	7319	556	22
19	7903	578	22	17	7901	578	22	31	7897	578	22

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SD	N22 - P1 P1	P1'	P1"	SD	N22 - P2 P2	P2'	P2"	SD	N22 - P3 P3	P3'	P3"
8	17			6	15			2	11		
7	43	26		5	41	26		10	37	26	
10	91	48	22	17	89	48	22	13	85	48	22
8	161	70	22	15	159	70	22	11	155	70	22
10	253	92	22	8	251	92	22	13	247	92	22
16	367	114	22	14	365	114	22	10	361	114	22
8	503	136	22	6	501	136	22	20	497	136	22
13	661	158	22	20	659	158	22	16	655	158	22
13	841	180	22	21	839	180	22	16	835	180	22
8	1043	202	22	6	1041	202	22	11	1037	202	22
16	1267	224	22	14	1265	224	22	10	1261	224	22
10	1513	246	22	8	1511	246	22	13	1507	246	22
17	1781	268	22	24	1779	268	22	20	1775	268	22
10	2071	290	22	17	2069	290	22	13	2065	290	22
16	2383	312	22	14	2381	312	22	19	2377	312	22
17	2717	334	22	15	2715	334	22	11	2711	334	22
13	3073	356	22	11	3071	356	22	16	3067	356	22
13	3451	378	22	20	3449	378	22	16	3445	378	22
17	3851	400	22	24	3849	400	22	20	3845	400	22
16	4273	422	22	14	4271	422	22	19	4267	422	22
19	4717	444	22	17	4715	444	22	13	4711	444	22
17	5183	466	22	15	5181	466	22	20	5177	466	22
19	5671	488	22	26	5669	488	22	22	5665	488	22
16	6181	510	22	23	6179	510	22	19	6175	510	22
17	6713	532	22	15	6711	532	22	20	6707	532	22
22	7267	554	22	20	7265	554	22	16	7261	554	22
22	7843	576	22	20	7841	576	22	25	7837	576	22

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SD	N22 - Q1 Q1	Q1'	Q1"	SD	N22 - Q2 Q2	Q2'	Q2"	SD	N22 - Q3 Q3	Q3'	Q3"
10	19			8	17			4	13		
7	43	24		5	41	24		10	37	24	
17	89	46	22	15	87	46	22	11	83	46	22
13	157	68	22	11	155	68	22	7	151	68	22
13	247	90	22	11	245	90	22	7	241	90	22
17	359	112	22	15	357	112	22	11	353	112	22
16	493	134	22	14	491	134	22	19	487	134	22
19	649	156	22	17	647	156	22	13	643	156	22
17	827	178	22	15	825	178	22	11	821	178	22
10	1027	200	22	8	1025	200	22	4	1021	200	22
16	1249	222	22	14	1247	222	22	10	1243	222	22
17	1493	244	22	15	1491	244	22	20	1487	244	22
22	1759	266	22	20	1757	266	22	16	1753	266	22
13	2047	288	22	11	2045	288	22	7	2041	288	22
17	2357	310	22	15	2355	310	22	11	2351	310	22
25	2689	332	22	23	2687	332	22	19	2683	332	22
10	3043	354	22	8	3041	354	22	13	3037	354	22
17	3419	376	22	15	3417	376	22	11	3413	376	22
19	3817	398	22	17	3815	398	22	13	3811	398	22
16	4237	420	22	14	4235	420	22	10	4231	420	22
26	4679	442	22	24	4677	442	22	20	4673	442	22
13	5143	464	22	11	5141	464	22	16	5137	464	22
22	5629	486	22	20	5627	486	22	16	5623	486	22
17	6137	508	22	15	6135	508	22	11	6131	508	22
25	6667	530	22	23	6665	530	22	19	6661	530	22
19	7219	552	22	17	7217	552	22	13	7213	552	22
26	7793	574	22	24	7791	574	22	29	7787	574	22

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SD	N22 - R1 R1	R1'	R1"	SD	N22 - R2 R2	R2'	R2"	SD	N22 - R3 R3	R3'	R3"
4	13			2	11			7	7		
8	35	22		6	33	22		11	29	22	
16	79	44	22	14	77	44	22	10	73	44	22
10	145	66	22	8	143	66	22	13	139	66	22
8	233	88	22	6	231	88	22	11	227	88	22
10	343	110	22	8	341	110	22	13	337	110	22
16	475	132	22	14	473	132	22	19	469	132	22
17	629	154	22	15	627	154	22	11	623	154	22
13	805	176	22	11	803	176	22	25	799	176	22
4	1003	198	22	2	1001	198	22	25	997	198	22
8	1223	220	22	6	1221	220	22	11	1217	220	22
16	1465	242	22	14	1463	242	22	19	1459	242	22
19	1729	264	22	17	1727	264	22	13	1723	264	22
8	2015	286	22	6	2013	286	22	11	2009	286	22
10	2323	308	22	8	2321	308	22	13	2317	308	22
16	2653	330	22	14	2651	330	22	19	2647	330	22
8	3005	352	22	6	3003	352	22	29	2999	352	22
22	3379	374	22	20	3377	374	22	16	3373	374	22
22	3775	396	22	20	3773	396	22	25	3769	396	22
17	4193	418	22	15	4191	418	22	20	4187	418	22
16	4633	440	22	14	4631	440	22	19	4627	440	22
19	5095	462	22	17	5093	462	22	22	5089	462	22
26	5579	484	22	24	5577	484	22	20	5573	484	22
19	6085	506	22	17	6083	506	22	22	6079	506	22
16	6613	528	22	14	6611	528	22	19	6607	528	22
17	7163	550	22	15	7161	550	22	20	7157	550	22
22	7735	572	22	20	7733	572	22	25	7729	572	22

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SD	N22 - S1 S1	S1'	S1"	SD	N22 - S2 S2	S2'	S2"	SD	N22 - S3 S3	S3'	S3"
2	11			9	9			5	5		
4	31	20		11	29	20		7	25	20	
10	73	42	22	8	71	42	22	13	67	42	22
11	137	64	22	9	135	64	22	5	131	64	22
7	223	86	22	5	221	86	22	10	217	86	22
7	331	108	22	14	329	108	22	10	325	108	22
11	461	130	22	18	459	130	22	14	455	130	22
10	613	152	22	8	611	152	22	13	607	152	22
22	787	174	22	20	785	174	22	16	781	174	22
20	983	196	22	18	981	196	22	23	977	196	22
4	1201	218	22	20	1199	218	22	16	1195	218	22
10	1441	240	22	17	1439	240	22	13	1435	240	22
11	1703	262	22	9	1701	262	22	23	1697	262	22
25	1987	284	22	23	1985	284	22	19	1981	284	22
16	2293	306	22	14	2291	306	22	19	2287	306	22
11	2621	328	22	18	2619	328	22	14	2615	328	22
19	2971	350	22	26	2969	350	22	22	2965	350	22
13	3343	372	22	11	3341	372	22	16	3337	372	22
20	3737	394	22	18	3735	394	22	14	3731	394	22
13	4153	416	22	11	4151	416	22	16	4147	416	22
19	4591	438	22	26	4589	438	22	22	4585	438	22
11	5051	460	22	18	5049	460	22	14	5045	460	22
16	5533	482	22	14	5531	482	22	20	5527	482	22
16	6037	504	22	14	6035	504	22	10	6031	504	22
20	6563	526	22	18	6561	526	22	23	6557	526	22
10	7111	548	22	17	7109	548	22	13	7105	548	22
22	7681	570	22	29	7679	570	22	25	7675	570	22

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SD	N22 - T1 T1	T1'	T1"	SD	N22 - T2 T2	T2'	T2"	SD	N22 - T3 T3	T3'	T3"
4	13			2	11			7	7		
4	31	18		11	29	18		7	25	18	
8	71	40	22	15	69	40	22	11	65	40	22
7	133	62	22	5	131	62	22	10	127	62	22
10	217	84	22	8	215	84	22	4	211	84	22
8	323	106	22	6	321	106	22	11	317	106	22
10	451	128	22	17	449	128	22	13	445	128	22
7	601	150	22	23	599	150	22	19	595	150	22
17	773	172	22	15	771	172	22	20	767	172	22
22	967	194	22	20	965	194	22	16	961	194	22
13	1183	216	22	11	1181	216	22	16	1177	216	22
8	1421	238	22	15	1419	238	22	11	1415	238	22
16	1681	260	22	23	1679	260	22	19	1675	260	22
19	1963	282	22	17	1961	282	22	22	1957	282	22
17	2267	304	22	15	2265	304	22	11	2261	304	22
19	2593	326	22	17	2591	326	22	22	2587	326	22
16	2941	348	22	23	2939	348	22	19	2935	348	22
8	3311	370	22	15	3309	370	22	11	3305	370	22
13	3703	392	22	11	3701	392	22	25	3697	392	22
13	4117	414	22	11	4115	414	22	7	4111	414	22
17	4553	436	22	15	4551	436	22	20	4547	436	22
7	5011	458	22	14	5009	458	22	10	5005	458	22
19	5491	480	22	26	5489	480	22	22	5485	480	22
26	5993	502	22	24	5991	502	22	29	5987	502	22
19	6517	524	22	17	6515	524	22	13	6511	524	22
16	7063	546	22	14	7061	546	22	19	7057	546	22
17	7631	568	22	24	7629	568	22	20	7625	568	22

- 4 → - 2 →

Table 6-B2 : Quadratic Polynomials of the Spiral-Graphs belonging to the "Prime-Number-Spiral-Systems" **P20-D** to **P20-I** and **N20-D** to **N20-I** (with the **2. Differential = 20**)

Spiral Graph System	Spiral Graph	Number Sequence of Spiral Graph	Quadratic Polynomial 1 (calculated with the first 3 numbers of the given sequence)	Quadratic Polynomial 2 (calculated with 3 numbers starting with the **2.** Number of the sequence)	Quadratic Polynomial 3 (calculated with 3 numbers starting with the **3.** Number of the sequence)	Quadratic Polynomial 4 (calculated with 3 numbers starting with the **4.** Number of the sequence)
N20-D	D3	1 , 21 , 61 , 121 , 201 , 301 ,......	$f_1(x) = 10x^2 - 10x + 1$	$f_2(x) = 10x^2 + 10x + 1$	$f_3(x) = 10x^2 + 30x + 21$	$f_4(x) = 10x^2 + 50x + 61$
	D2	5 , 25 , 65 , 125 , 205 , 305 ,......	$f_1(x) = 10x^2 - 10x + 5$	$f_2(x) = 10x^2 + 10x + 5$	$f_3(x) = 10x^2 + 30x + 25$	$f_4(x) = 10x^2 + 50x + 65$
	D1	7 , 27 , 67 , 127 , 207 , 307 ,......	$f_1(x) = 10x^2 - 10x + 7$	$f_2(x) = 10x^2 + 10x + 7$	$f_3(x) = 10x^2 + 30x + 27$	$f_4(x) = 10x^2 + 50x + 67$
P20-D	D1	11 , 31 , 71 , 131 , 211 , 311 ,......	$f_1(x) = 10x^2 - 10x + 11$	$f_2(x) = 10x^2 + 10x + 11$	$f_3(x) = 10x^2 + 30x + 31$	$f_4(x) = 10x^2 + 50x + 71$
	D2	13 , 33 , 73 , 133 , 213 , 313 ,......	$f_1(x) = 10x^2 - 10x + 13$	$f_2(x) = 10x^2 + 10x + 13$	$f_3(x) = 10x^2 + 30x + 33$	$f_4(x) = 10x^2 + 50x + 73$
	D3	17 , 37 , 77 , 137 , 217 , 317 ,......	$f_1(x) = 10x^2 - 10x + 17$	$f_2(x) = 10x^2 + 10x + 17$	$f_3(x) = 10x^2 + 30x + 37$	$f_4(x) = 10x^2 + 50x + 77$
N20-E	E3	1 , 25 , 69 , 133 , 217 , 321 ,......	$f_1(x) = 10x^2 - 6x - 3$	$f_2(x) = 10x^2 + 14x + 1$	$f_3(x) = 10x^2 + 34x + 25$	$f_4(x) = 10x^2 + 54x + 69$
	E2	1 , 5 , 29 , 73 , 137 , 221 ,......	$f_1(x) = 10x^2 - 26x + 17$	$f_2(x) = 10x^2 - 6x + 1$	$f_3(x) = 10x^2 + 14x + 5$	$f_4(x) = 10x^2 + 34x + 29$
	E1	3 , 7 , 31 , 75 , 139 , 223 ,......	$f_1(x) = 10x^2 - 26x + 19$	$f_2(x) = 10x^2 - 6x + 3$	$f_3(x) = 10x^2 + 14x + 7$	$f_4(x) = 10x^2 + 34x + 31$
P20-E	E1	7 , 11 , 35 , 79 , 143 , 227 ,......	$f_1(x) = 10x^2 - 26x + 23$	$f_2(x) = 10x^2 - 6x + 7$	$f_3(x) = 10x^2 + 14x + 11$	$f_4(x) = 10x^2 + 34x + 35$
	E2	9 , 13 , 37 , 81 , 145 , 229 ,......	$f_1(x) = 10x^2 - 26x + 25$	$f_2(x) = 10x^2 - 6x + 9$	$f_3(x) = 10x^2 + 14x + 13$	$f_4(x) = 10x^2 + 34x + 37$
	E3	13 , 17 , 41 , 85 , 149 , 233 ,......	$f_1(x) = 10x^2 - 26x + 29$	$f_2(x) = 10x^2 - 6x + 13$	$f_3(x) = 10x^2 + 14x + 17$	$f_4(x) = 10x^2 + 34x + 41$
N20-F	F3	1 , 27 , 73 , 139 , 225 , 331 ,......	$f_1(x) = 10x^2 - 4x - 5$	$f_2(x) = 10x^2 + 16x + 1$	$f_3(x) = 10x^2 + 36x + 27$	$f_4(x) = 10x^2 + 56x + 73$
	F2	5 , 31 , 77 , 143 , 229 , 335 ,......	$f_1(x) = 10x^2 - 4x - 1$	$f_2(x) = 10x^2 + 16x + 5$	$f_3(x) = 10x^2 + 36x + 31$	$f_4(x) = 10x^2 + 56x + 77$
	F1	7 , 33 , 79 , 145 , 231 , 337 ,......	$f_1(x) = 10x^2 - 4x + 1$	$f_2(x) = 10x^2 + 16x + 7$	$f_3(x) = 10x^2 + 36x + 33$	$f_4(x) = 10x^2 + 56x + 79$
P20-F	F1	11 , 37 , 83 , 149 , 235 , 341 ,......	$f_1(x) = 10x^2 - 4x + 5$	$f_2(x) = 10x^2 + 16x + 11$	$f_3(x) = 10x^2 + 36x + 37$	$f_4(x) = 10x^2 + 56x + 83$
	F2	13 , 39 , 85 , 151 , 237 , 343 ,......	$f_1(x) = 10x^2 - 4x + 7$	$f_2(x) = 10x^2 + 16x + 13$	$f_3(x) = 10x^2 + 36x + 39$	$f_4(x) = 10x^2 + 56x + 85$
	F3	17 , 43 , 89 , 155 , 241 , 347 ,......	$f_1(x) = 10x^2 - 4x + 11$	$f_2(x) = 10x^2 + 16x + 17$	$f_3(x) = 10x^2 + 36x + 43$	$f_4(x) = 10x^2 + 56x + 89$
N20-G	G3	3 , 13 , 43 , 93 , 163 , 253 ,......	$f_1(x) = 10x^2 - 20x + 13$	$f_2(x) = 10x^2 + 0x + 3$	$f_3(x) = 10x^2 + 20x + 13$	$f_4(x) = 10x^2 + 40x + 43$
	G2	7 , 17 , 47 , 97 , 167 , 257 ,......	$f_1(x) = 10x^2 - 20x + 17$	$f_2(x) = 10x^2 + 0x + 7$	$f_3(x) = 10x^2 + 20x + 17$	$f_4(x) = 10x^2 + 40x + 47$
	G1	9 , 19 , 49 , 99 , 169 , 259 ,......	$f_1(x) = 10x^2 - 20x + 19$	$f_2(x) = 10x^2 + 0x + 9$	$f_3(x) = 10x^2 + 20x + 19$	$f_4(x) = 10x^2 + 40x + 49$
P20-G	G1	13 , 23 , 53 , 103 , 173 , 263 ,......	$f_1(x) = 10x^2 - 20x + 23$	$f_2(x) = 10x^2 + 0x + 13$	$f_3(x) = 10x^2 + 20x + 23$	$f_4(x) = 10x^2 + 40x + 53$
	G2	15 , 25 , 55 , 105 , 175 , 265 ,......	$f_1(x) = 10x^2 - 20x + 25$	$f_2(x) = 10x^2 + 0x + 15$	$f_3(x) = 10x^2 + 20x + 25$	$f_4(x) = 10x^2 + 40x + 55$
	G3	19 , 29 , 59 , 109 , 179 , 269 ,......	$f_1(x) = 10x^2 - 20x + 29$	$f_2(x) = 10x^2 + 0x + 19$	$f_3(x) = 10x^2 + 20x + 29$	$f_4(x) = 10x^2 + 40x + 59$
N20-H	H3	1 , 15 , 49 , 103 , 177 , 271 ,......	$f_1(x) = 10x^2 - 16x + 7$	$f_2(x) = 10x^2 + 4x + 1$	$f_3(x) = 10x^2 + 24x + 15$	$f_4(x) = 10x^2 + 44x + 49$
	H2	5 , 19 , 53 , 107 , 181 , 275 ,......	$f_1(x) = 10x^2 - 16x + 11$	$f_2(x) = 10x^2 + 4x + 5$	$f_3(x) = 10x^2 + 24x + 19$	$f_4(x) = 10x^2 + 44x + 53$
	H1	7 , 21 , 55 , 109 , 183 , 277 ,......	$f_1(x) = 10x^2 - 16x + 13$	$f_2(x) = 10x^2 + 4x + 7$	$f_3(x) = 10x^2 + 24x + 21$	$f_4(x) = 10x^2 + 44x + 55$
P20-H	H1	11 , 25 , 59 , 113 , 187 , 281 ,......	$f_1(x) = 10x^2 - 16x + 17$	$f_2(x) = 10x^2 + 4x + 11$	$f_3(x) = 10x^2 + 24x + 25$	$f_4(x) = 10x^2 + 44x + 59$
	H2	13 , 27 , 61 , 115 , 189 , 283 ,......	$f_1(x) = 10x^2 - 16x + 19$	$f_2(x) = 10x^2 + 4x + 13$	$f_3(x) = 10x^2 + 24x + 27$	$f_4(x) = 10x^2 + 44x + 61$
	H3	17 , 31 , 65 , 119 , 193 , 287 ,......	$f_1(x) = 10x^2 - 16x + 23$	$f_2(x) = 10x^2 + 4x + 17$	$f_3(x) = 10x^2 + 24x + 31$	$f_4(x) = 10x^2 + 44x + 65$
N20-I	I3	3 , 19 , 55 , 111 , 187 , 283 ,......	$f_1(x) = 10x^2 - 14x + 7$	$f_2(x) = 10x^2 + 6x + 3$	$f_3(x) = 10x^2 + 26x + 19$	$f_4(x) = 10x^2 + 46x + 55$
	I2	7 , 23 , 59 , 115 , 191 , 287 ,......	$f_1(x) = 10x^2 - 14x + 11$	$f_2(x) = 10x^2 + 6x + 7$	$f_3(x) = 10x^2 + 26x + 23$	$f_4(x) = 10x^2 + 46x + 59$
	I1	9 , 25 , 61 , 117 , 193 , 289 ,......	$f_1(x) = 10x^2 - 14x + 13$	$f_2(x) = 10x^2 + 6x + 9$	$f_3(x) = 10x^2 + 26x + 25$	$f_4(x) = 10x^2 + 46x + 61$
P20-I	I1	13 , 29 , 65 , 121 , 197 , 293 ,......	$f_1(x) = 10x^2 - 14x + 17$	$f_2(x) = 10x^2 + 6x + 13$	$f_3(x) = 10x^2 + 26x + 29$	$f_4(x) = 10x^2 + 46x + 65$
	I2	15 , 31 , 67 , 123 , 199 , 295 ,......	$f_1(x) = 10x^2 - 14x + 19$	$f_2(x) = 10x^2 + 6x + 15$	$f_3(x) = 10x^2 + 26x + 31$	$f_4(x) = 10x^2 + 46x + 67$
	I3	19 , 35 , 71 , 127 , 203 , 299 ,......	$f_1(x) = 10x^2 - 14x + 23$	$f_2(x) = 10x^2 + 6x + 19$	$f_3(x) = 10x^2 + 26x + 35$	$f_4(x) = 10x^2 + 46x + 71$

3 , 3 , 2 , 1

ble 6-C2 : Quadratic Polynomials of the Spiral-Graphs belonging to the "Prime-Number-Spiral-Systems" **N22-J** to **N22-T** (with the **2. Differential = 22**)

piral Graph System	**Spiral Graph**	**Number Sequence of Spiral Graph**	**Quadratic Polynomial 1** (calculated with the first 3 numbers of the given sequence)	**Quadratic Polynomial 2** (calculated with 3 numbers starting with the **2.** Number of the sequence)	**Quadratic Polynomial 3** (calculated with 3 numbers starting with the **3.** Number of the sequence)	**Quadratic Polynomial 4** (calculated with 3 numbers starting with the **4.** Number of the sequence)
N22-J	J1	15 , 31 , 69 , 129 , 211 , 315 ,......	$f_1(x) = 11x^2 - 17x + 21$	$f_2(x) = 11x^2 + 5x + 15$	$f_3(x) = 11x^2 + 27x + 31$	$f_4(x) = 11x^2 + 49x + 69$
	J2	13 , 29 , 67 , 127 , 209 , 313 ,......	$f_1(x) = 11x^2 - 17x + 19$	$f_2(x) = 11x^2 + 5x + 13$	$f_3(x) = 11x^2 + 27x + 29$	$f_4(x) = 11x^2 + 49x + 67$
	J3	9 , 25 , 63 , 123 , 205 , 309 ,......	$f_1(x) = 11x^2 - 17x + 15$	$f_2(x) = 11x^2 + 5x + 9$	$f_3(x) = 11x^2 + 27x + 25$	$f_4(x) = 11x^2 + 49x + 63$
N22-K	K1	11 , 25 , 61 , 119 , 199 , 301 ,......	$f_1(x) = 11x^2 - 19x + 19$	$f_2(x) = 11x^2 + 3x + 11$	$f_3(x) = 11x^2 + 25x + 25$	$f_4(x) = 11x^2 + 47x + 61$
	K2	9 , 23 , 59 , 117 , 197 , 299 ,......	$f_1(x) = 11x^2 - 19x + 17$	$f_2(x) = 11x^2 + 3x + 9$	$f_3(x) = 11x^2 + 25x + 23$	$f_4(x) = 11x^2 + 47x + 59$
	K3	5 , 19 , 55 , 113 , 193 , 295 ,......	$f_1(x) = 11x^2 - 19x + 13$	$f_2(x) = 11x^2 + 3x + 5$	$f_3(x) = 11x^2 + 25x + 19$	$f_4(x) = 11x^2 + 47x + 55$
N22-L	L1	13 , 25 , 59 , 115 , 193 , 293 ,......	$f_1(x) = 11x^2 - 21x + 23$	$f_2(x) = 11x^2 + 1x + 13$	$f_3(x) = 11x^2 + 23x + 25$	$f_4(x) = 11x^2 + 45x + 59$
	L2	11 , 23 , 57 , 113 , 191 , 291 ,......	$f_1(x) = 11x^2 - 21x + 21$	$f_2(x) = 11x^2 + 1x + 11$	$f_3(x) = 11x^2 + 23x + 23$	$f_4(x) = 11x^2 + 45x + 57$
	L3	7 , 19 , 53 , 109 , 187 , 287 ,......	$f_1(x) = 11x^2 - 21x + 17$	$f_2(x) = 11x^2 + 1x + 7$	$f_3(x) = 11x^2 + 23x + 19$	$f_4(x) = 11x^2 + 45x + 53$
N22-M	M1	13 , 23 , 55 , 109 , 185 , 283 ,......	$f_1(x) = 11x^2 - 23x + 25$	$f_2(x) = 11x^2 - 1x + 13$	$f_3(x) = 11x^2 + 21x + 23$	$f_4(x) = 11x^2 + 43x + 55$
	M2	11 , 21 , 53 , 107 , 183 , 281 ,......	$f_1(x) = 11x^2 - 23x + 23$	$f_2(x) = 11x^2 - 1x + 11$	$f_3(x) = 11x^2 + 21x + 21$	$f_4(x) = 11x^2 + 43x + 53$
	M3	7 , 17 , 49 , 103 , 179 , 277 ,......	$f_1(x) = 11x^2 - 23x + 19$	$f_2(x) = 11x^2 - 1x + 7$	$f_3(x) = 11x^2 + 21x + 17$	$f_4(x) = 11x^2 + 43x + 49$
N22-N	N1	11 , 19 , 49 , 101 , 175 , 271 ,......	$f_1(x) = 11x^2 - 25x + 25$	$f_2(x) = 11x^2 - 3x + 11$	$f_3(x) = 11x^2 + 19x + 19$	$f_4(x) = 11x^2 + 41x + 49$
	N2	9 , 17 , 47 , 99 , 173 , 269 ,......	$f_1(x) = 11x^2 - 25x + 23$	$f_2(x) = 11x^2 - 3x + 9$	$f_3(x) = 11x^2 + 19x + 17$	$f_4(x) = 11x^2 + 41x + 47$
	N3	5 , 13 , 43 , 95 , 169 , 265 ,......	$f_1(x) = 11x^2 - 25x + 19$	$f_2(x) = 11x^2 - 3x + 5$	$f_3(x) = 11x^2 + 19x + 13$	$f_4(x) = 11x^2 + 41x + 43$
N22-O	O1	19 , 25 , 53 , 103 , 175 , 269 ,......	$f_1(x) = 11x^2 - 27x + 35$	$f_2(x) = 11x^2 - 5x + 19$	$f_3(x) = 11x^2 + 17x + 25$	$f_4(x) = 11x^2 + 39x + 53$
	O2	17 , 23 , 51 , 101 , 173 , 267 ,......	$f_1(x) = 11x^2 - 27x + 33$	$f_2(x) = 11x^2 - 5x + 17$	$f_3(x) = 11x^2 + 17x + 23$	$f_4(x) = 11x^2 + 39x + 51$
	O3	13 , 19 , 47 , 97 , 169 , 263 ,......	$f_1(x) = 11x^2 - 27x + 29$	$f_2(x) = 11x^2 - 5x + 13$	$f_3(x) = 11x^2 + 17x + 19$	$f_4(x) = 11x^2 + 39x + 47$
N22-P	P1	13 , 17 , 43 , 91 , 161 , 253 ,......	$f_1(x) = 11x^2 - 29x + 31$	$f_2(x) = 11x^2 - 7x + 13$	$f_3(x) = 11x^2 + 15x + 17$	$f_4(x) = 11x^2 + 37x + 43$
	P2	11 , 15 , 41 , 89 , 159 , 251 ,......	$f_1(x) = 11x^2 - 29x + 29$	$f_2(x) = 11x^2 - 7x + 11$	$f_3(x) = 11x^2 + 15x + 15$	$f_4(x) = 11x^2 + 37x + 41$
	P3	7 , 11 , 37 , 85 , 155 , 247 ,......	$f_1(x) = 11x^2 - 29x + 25$	$f_2(x) = 11x^2 - 7x + 7$	$f_3(x) = 11x^2 + 15x + 11$	$f_4(x) = 11x^2 + 37x + 37$
N22-Q	Q1	17 , 19 , 43 , 89 , 157 , 247 ,......	$f_1(x) = 11x^2 - 31x + 37$	$f_2(x) = 11x^2 - 9x + 17$	$f_3(x) = 11x^2 + 13x + 19$	$f_4(x) = 11x^2 + 35x + 43$
	Q2	15 , 17 , 41 , 87 , 155 , 245 ,......	$f_1(x) = 11x^2 - 31x + 35$	$f_2(x) = 11x^2 - 9x + 15$	$f_3(x) = 11x^2 + 13x + 17$	$f_4(x) = 11x^2 + 35x + 41$
	Q3	11 , 13 , 37 , 83 , 151 , 241 ,......	$f_1(x) = 11x^2 - 31x + 31$	$f_2(x) = 11x^2 - 9x + 11$	$f_3(x) = 11x^2 + 13x + 13$	$f_4(x) = 11x^2 + 35x + 37$
N22-R	R1	13 , 35 , 79 , 145 , 233 , 343 ,......	$f_1(x) = 11x^2 - 11x + 13$	$f_2(x) = 11x^2 + 11x + 13$	$f_3(x) = 11x^2 + 33x + 35$	$f_4(x) = 11x^2 + 55x + 79$
	R2	11 , 33 , 77 , 143 , 231 , 341 ,......	$f_1(x) = 11x^2 - 11x + 11$	$f_2(x) = 11x^2 + 11x + 11$	$f_3(x) = 11x^2 + 33x + 33$	$f_4(x) = 11x^2 + 55x + 77$
	R3	7 , 29 , 73 , 139 , 227 , 337 ,......	$f_1(x) = 11x^2 - 11x + 7$	$f_2(x) = 11x^2 + 11x + 7$	$f_3(x) = 11x^2 + 33x + 29$	$f_4(x) = 11x^2 + 55x + 73$
N22-S	S1	11 , 31 , 73 , 137 , 223 , 331 ,......	$f_1(x) = 11x^2 - 13x + 13$	$f_2(x) = 11x^2 + 9x + 11$	$f_3(x) = 11x^2 + 31x + 31$	$f_4(x) = 11x^2 + 53x + 73$
	S2	9 , 29 , 71 , 135 , 221 , 329 ,......	$f_1(x) = 11x^2 - 13x + 11$	$f_2(x) = 11x^2 + 9x + 9$	$f_3(x) = 11x^2 + 31x + 29$	$f_4(x) = 11x^2 + 53x + 71$
	S3	5 , 25 , 67 , 131 , 217 , 325 ,......	$f_1(x) = 11x^2 - 13x + 7$	$f_2(x) = 11x^2 + 9x + 5$	$f_3(x) = 11x^2 + 31x + 25$	$f_4(x) = 11x^2 + 53x + 67$
N22-T	T1	13 , 31 , 71 , 133 , 217 , 323 ,......	$f_1(x) = 11x^2 - 15x + 17$	$f_2(x) = 11x^2 + 7x + 13$	$f_3(x) = 11x^2 + 29x + 31$	$f_4(x) = 11x^2 + 51x + 71$
	T2	11 , 29 , 69 , 131 , 215 , 321 ,......	$f_1(x) = 11x^2 - 15x + 15$	$f_2(x) = 11x^2 + 7x + 11$	$f_3(x) = 11x^2 + 29x + 29$	$f_4(x) = 11x^2 + 51x + 69$
	T3	7 , 25 , 65 , 127 , 211 , 317 ,......	$f_1(x) = 11x^2 - 15x + 11$	$f_2(x) = 11x^2 + 7x + 7$	$f_3(x) = 11x^2 + 29x + 25$	$f_4(x) = 11x^2 + 51x + 65$